The field content of the Minimal Supersymmetric Standard Model (MSSM)

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	Vector supermultiplets	Superfield	Adj. repr.	Spin-1 (gauge bosons)	Spin-1/2 (gauginos)	Aux.	Vector superfield (in Wess-Zumino gauge)
Gauge fields	$\mathrm{U}(1)_{\mathrm{Y}}$	V_{Y}	$\mathbf{1},\mathbf{1},0$	B_μ, B-boson	$\lambda_{\mathrm{Y}} \equiv \widetilde{B}$, bino	D_{Y}	$V_{\mathrm{Y}} \equiv \theta\,\sigma^\mu\,\bar\theta\,B_\mu + \theta\theta\,\bar\theta\bar\lambda_{\mathrm{Y}} + \bar\theta\bar\theta\,\theta\lambda_{\mathrm{Y}} + \frac{1}{2}\theta\theta\,\bar\theta\bar\theta\,D_{\mathrm{Y}}$
	$\mathrm{SU}(2)_{\mathrm{L}}$	V^i_{L}	$\mathbf{1},\mathbf{3},0$	W^i_μ, W-bosons	$\lambda^i_{\mathrm{L}} \equiv \widetilde{W}^i$, winos	D^i_{L}	$V^i_{\mathrm{L}} \equiv \theta\,\sigma^\mu\,\bar\theta\,W^i_\mu + \theta\theta\,\bar\theta\bar\lambda^i_{\mathrm{L}} + \bar\theta\bar\theta\,\theta\lambda^i_{\mathrm{L}} + \frac{1}{2}\theta\theta\,\bar\theta\bar\theta\,D^i_{\mathrm{L}}$
	$\mathrm{SU}(3)_{\mathrm{C}}$	V^a_{C}	$\mathbf{8},\mathbf{1},0$	g^a_μ, gluons	$\lambda^a_{\mathrm{C}} \equiv \widetilde{g}^a$, gluinos	D^a_{C}	$V^a_{\mathrm{C}} \equiv \theta\,\sigma^\mu\,\bar\theta\,g^a_\mu + \theta\theta\,\bar\theta\bar\lambda^a_{\mathrm{C}} + \bar\theta\bar\theta\,\theta\lambda^a_{\mathrm{C}} + \frac{1}{2}\theta\theta\,\bar\theta\bar\theta\,D^a_{\mathrm{C}}$

	Chiral supermultiplets	Superfield	Repr.	Spin-1/2 (fermions)	Spin-0 (sfermions)	Aux.	Chiral superfield (in terms of $y^\mu = x^\mu - \mathrm{i}\,\theta\sigma^\mu\bar\theta$)
Matter fields	quarks, s(calar) quarks	Q_I	$\mathbf{3},\mathbf{2},+\frac{1}{6}$	$\begin{pmatrix} u_{\mathrm{L}} \\ d_{\mathrm{L}} \end{pmatrix}, \begin{pmatrix} \chi_u \\ \chi_d \end{pmatrix}$	$\begin{pmatrix} \widetilde{u}_{\mathrm{L}} \\ \widetilde{d}_{\mathrm{L}} \end{pmatrix}, \begin{pmatrix} \widetilde{\phi}_u \\ \widetilde{\phi}_d \end{pmatrix}$	$\begin{pmatrix} F_u \\ F_d \end{pmatrix}$	$Q = Q_1 = \begin{pmatrix} \widetilde{\phi}_u + \sqrt{2}\,\theta\chi_u + \theta\theta\,F_u \\ \widetilde{\phi}_d + \sqrt{2}\,\theta\chi_d + \theta\theta\,F_d \end{pmatrix}$
		$U_I\ (\bar u_I)$	$\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3}$	$\bar u_{\mathrm{L}} = (u_{\mathrm{R}})^{\mathrm{C}}$, $\chi_{\bar u}$	$\widetilde{\bar u}_{\mathrm{L}}$, $\widetilde{\phi}_{\bar u}$	$F_{\bar u}$	$\bar u = U_1 = \widetilde{\phi}_{\bar u} + \sqrt{2}\,\theta\chi_{\bar u} + \theta\theta\,F_{\bar u}$
		$D_I\ (\bar d_I)$	$\bar{\mathbf{3}},\mathbf{1},+\frac{1}{3}$	$\bar d_{\mathrm{L}} = (d_{\mathrm{R}})^{\mathrm{C}}$, $\chi_{\bar d}$	$\widetilde{\bar d}_{\mathrm{L}}$, $\widetilde{\phi}_{\bar d}$	$F_{\bar d}$	$\bar d = D_1 = \widetilde{\phi}_{\bar d} + \sqrt{2}\,\theta\chi_{\bar d} + \theta\theta\,F_{\bar d}$
	leptons, s(calar) leptons	L_I	$\mathbf{1},\mathbf{2},-\frac{1}{2}$	$\begin{pmatrix} \nu_{e\mathrm{L}} \\ e_{\mathrm{L}} \end{pmatrix}, \begin{pmatrix} \chi_{\nu_e} \\ \chi_e \end{pmatrix}$	$\begin{pmatrix} \widetilde{\nu}_{e\mathrm{L}} \\ \widetilde{e}_{\mathrm{L}} \end{pmatrix}, \begin{pmatrix} \widetilde{\phi}_{\nu_e} \\ \widetilde{\phi}_e \end{pmatrix}$	$\begin{pmatrix} F_{\nu_e} \\ F_e \end{pmatrix}$	$L = L_1 = \begin{pmatrix} \widetilde{\phi}_{\nu_e} + \sqrt{2}\,\theta\chi_{\nu_e} + \theta\theta\,F_{\nu_e} \\ \widetilde{\phi}_e + \sqrt{2}\,\theta\chi_e + \theta\theta\,F_e \end{pmatrix}$
		$N_I\ (\bar\nu_I)$	$\mathbf{1},\mathbf{1},0$	$\bar\nu_{e\mathrm{L}} = (\nu_{e\mathrm{R}})^{\mathrm{C}}$, $\chi_{\bar\nu_e}$	$\widetilde{\bar\nu}_{e\mathrm{L}}$, $\widetilde{\phi}_{\bar\nu_e}$	$F_{\bar\nu_e}$	$\bar\nu = N_1 = \widetilde{\phi}_{\bar\nu_e} + \sqrt{2}\,\theta\chi_{\bar\nu_e} + \theta\theta\,F_{\bar\nu_e}$
		$E_I\ (\bar e_I)$	$\mathbf{1},\mathbf{1},+1$	$\bar e_{\mathrm{L}} = (e_{\mathrm{R}})^{\mathrm{C}}$, $\chi_{\bar e}$	$\widetilde{\bar e}_{\mathrm{L}}$, $\widetilde{\phi}_{\bar e}$	$F_{\bar e}$	$\bar e = E_1 = \widetilde{\phi}_{\bar e} + \sqrt{2}\,\theta\chi_{\bar e} + \theta\theta\,F_{\bar e}$
Higgs fields	higgsinos, higgs	H_u	$\mathbf{1},\mathbf{2},+\frac{1}{2}$	$\begin{pmatrix} \widetilde{H}^+_u \\ \widetilde{H}^0_u \end{pmatrix}$	$\begin{pmatrix} H^+_u \\ H^0_u \end{pmatrix}$	$\begin{pmatrix} F^+_{H_u} \\ F^0_{H_u} \end{pmatrix}$	$H_u = \begin{pmatrix} H^+_u + \sqrt{2}\,\theta\widetilde{H}^+_u + \theta\theta\,F^+_{H_u} \\ H^0_u + \sqrt{2}\,\theta\widetilde{H}^0_u + \theta\theta\,F^0_{H_u} \end{pmatrix}$
		H_d	$\mathbf{1},\mathbf{2},-\frac{1}{2}$	$\begin{pmatrix} \widetilde{H}^0_d \\ \widetilde{H}^-_d \end{pmatrix}$	$\begin{pmatrix} H^0_d \\ H^-_d \end{pmatrix}$	$\begin{pmatrix} F^+_{H_d} \\ F^0_{H_d} \end{pmatrix}$	$H_d = \begin{pmatrix} H^0_d + \sqrt{2}\,\theta\widetilde{H}^0_d + \theta\theta\,F^+_{H_d} \\ H^-_d + \sqrt{2}\,\theta\widetilde{H}^-_d + \theta\theta\,F^0_{H_d} \end{pmatrix}$

N.B. Quantum numbers of the fields in the MSSM are the same as in the SM.

Superspace Formalism

A **vector superfield**, $\mathrm{SU}(n)$, in WZ gauge:

$$V^i(x,\theta,\bar\theta) \equiv \theta\sigma^\mu\bar\theta\,V^i_\mu(x) + \theta\theta\,\bar\theta\bar\lambda^i(x) + \bar\theta\bar\theta\,\theta\lambda^i(x) + \tfrac{1}{2}\theta\theta\,\bar\theta\bar\theta\,D^i(x), \quad V \equiv V^i T^i$$

The **field strength superfield**:

$$\mathcal{V}_\alpha \equiv \mathcal{V}^i_\alpha T^i \equiv -\tfrac{1}{4}\bar D_{\dot\beta}\bar D^{\dot\beta}\left(\mathrm{e}^{-2gV}\,D_\alpha\,\mathrm{e}^{2gV}\right)$$

The gauge kinetic term:

$$\tfrac{1}{4}(\mathcal{V}^{i\,\alpha}\mathcal{V}^i_\alpha)\big|_F + \text{h.c.}$$

$$\tfrac{1}{4}(\mathcal{V}^{i\,\alpha}\mathcal{V}^i_\alpha)\big|_F \ \ (\text{mod tot. der.}) = \tfrac{1}{2}D^iD^i + \mathrm{i}\bar\lambda^i\bar\sigma^\mu D_\mu\lambda^i - \tfrac{1}{4}F^i_{\mu\nu}F^{i\,\mu\nu}$$

The chiral covariant derivatives:

$$D_\alpha \equiv \partial_\alpha - \mathrm{i}\,\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\partial_\mu,$$
$$D^\alpha \equiv -\partial^\alpha + \mathrm{i}\,\bar\theta_{\dot\alpha}\bar\sigma^{\mu\,\dot\alpha\alpha}\,\partial_\mu,$$
$$\bar D^{\dot\alpha} \equiv \bar\partial^{\dot\alpha} - \mathrm{i}\,\theta_{\alpha}\sigma^{\mu\,\dot\alpha\alpha}\,\partial_\mu,$$
$$\bar D_{\dot\alpha} \equiv -\bar\partial_{\dot\alpha} + \mathrm{i}\,\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu$$

Observe:
$$\partial^\alpha = -\varepsilon^{\alpha\beta}\partial_\beta \qquad \bar\partial^{\dot\alpha} = -\varepsilon^{\dot\alpha\dot\beta}\bar\partial_{\dot\beta}$$

A **chiral superfield**:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\,\theta\chi(y) + \theta\theta\,F(y)$$

where $y^\mu = x^\mu - \mathrm{i}\,\theta\sigma^\mu\bar\theta$. (*) Expanded:

$$\Phi(x,\theta,\bar\theta) = \phi(x) + \sqrt{2}\,\theta\chi(x) + \theta\theta\,F(x) - \tfrac{1}{4}\,\theta\theta\,\bar\theta\bar\theta\,\Box\phi(x) - \tfrac{\mathrm{i}}{\sqrt{2}}\,\theta\theta\,\bar\theta\bar\sigma^\mu\partial_\mu\chi(x)$$

The canonical and gauge inv. kinetic terms:

free: $(\Phi^\dagger\Phi)\big|_D$, matter: $(\Phi^\dagger\,\mathrm{e}^{2gV}\,\Phi)\big|_D$

$$(\Phi^\dagger\Phi)\big|_D \ \ (\text{mod tot. der.}) = \partial_\mu\phi^\dagger\,\partial^\mu\phi + \mathrm{i}\bar\chi\bar\sigma^\mu\partial_\mu\chi + F^\dagger F$$

$$\mathrm{U}(1)\text{: } (\Phi^\dagger\,\mathrm{e}^{2gV}\,\Phi)\big|_D \ \ (\text{mod tot. der.}) = (D_\mu\phi)^\dagger\,D^\mu\phi + \mathrm{i}\bar\chi\bar\sigma^\mu D_\mu\chi + F^\dagger F - g\,\phi^\dagger\phi\,D - (\sqrt{2}g\,\chi\lambda\,\phi^\dagger + \text{h.c.})$$

(*) y are complex composite bosonic coordinates.

The **superpotential** for the collection $\{\Phi_I\}$:

$$\mathcal{W}(\Phi_I) = \tfrac{1}{2}m^{IJ}\Phi_I\Phi_J + \tfrac{1}{6}y^{IJK}\Phi_I\Phi_J\Phi_K$$

Opt. F-term (as $c^I\Phi_I$ in the superpotential):

$-k\Phi\big|_F = -kF$ (not in MSSM)

The interaction term:

$$\mathcal{W}(\Phi_I)\big|_F + \text{h.c.}$$

$$\mathcal{W}(\Phi_I)\big|_F + \text{h.c.} = \tfrac{\partial}{\partial\phi_I}\mathcal{W}(\phi_I)\,F_I - \tfrac{1}{2}\tfrac{\partial^2}{\partial\phi_I\partial\phi_J}\mathcal{W}(\phi_I)\,\chi_I\chi_J + \text{h.c.}$$

Opt. Fayet-Iliopoulos D-term (in U(1) only)

$-2\kappa V\big|_D = -\kappa D$ (not in MSSM)

The MSSM Lagrangian

The (matter) kinetic term

$$\mathcal{L}^{\text{kin}} = \sum_{\Phi,\,gV} \Phi^\dagger\,\mathrm{e}^{2gV}\,\Phi\big|_D$$

gV runs over all the gauge superfields, $\{\,gV_{\mathrm{Y}},\ g_iV^i_{\mathrm{L}}t^i,\ g_aV^a_{\mathrm{C}}T^a\,\}$,

Φ runs over all the matter superfields, $\{Q_I,\,U_I,\,D_I,\,L_I,\,N_I,\,E_I,\,H_u,\,H_d\}$,

in the appropriate respresentation where $I = 1,2,3$ is the family index.

The gauge kinetic term

$$\mathcal{L}^{\text{g-kin}} = \sum_{\mathcal{V}} \frac{1}{4}(\mathcal{V}^\alpha\mathcal{V}_\alpha)\big|_F + \text{h.c.}$$

$\mathcal{V}$ runs over all the field strength superfields

The Yukawa interaction term

$$\mathcal{L}^{\text{Yuk}} = \mathcal{W}(\Phi)\big|_F + \text{h.c.}$$

$\mathcal{W}$ is the superpotential (given below)

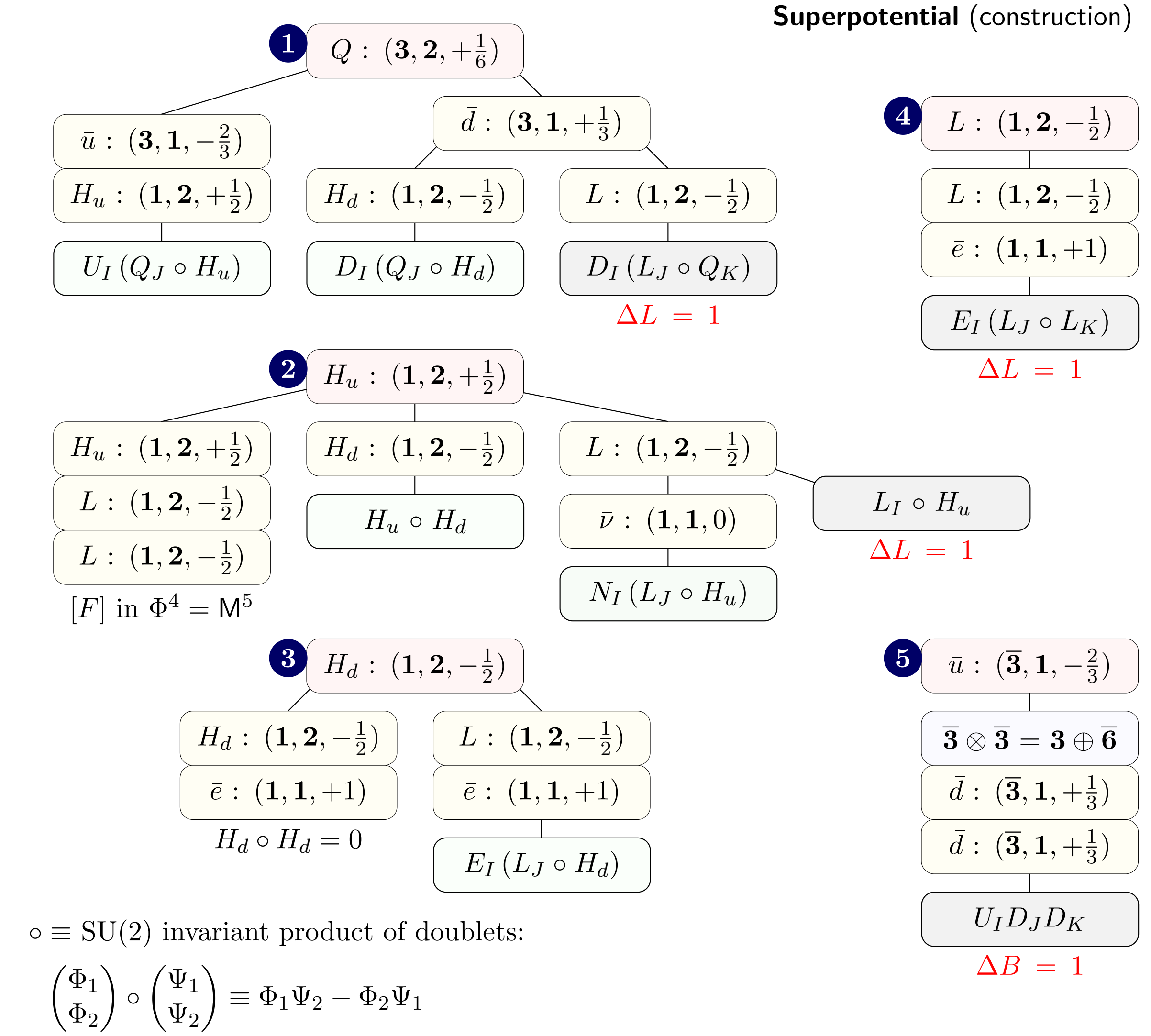
The Superpotential, Part I ('good' terms)

$$\mathcal{W}_1 = y^{IJ}_u\,U_I\,(Q_J\circ H_u) - y^{IJ}_d\,D_I\,(Q_J\circ H_d) + y^{IJ}_\nu\,N_I\,(L_J\circ H_u) - y^{IJ}_e\,E_I\,(L_J\circ H_d) + \mu\,(H_u\circ H_d)$$

The Superpotential, Part II (terms that violate lepton/baryon numbers)

$$\mathcal{W}_2 = \lambda^{IJK}\,E_I\,(L_J\circ L_K) + \lambda'^{IJK}\,D_I\,(L_J\circ Q_K) + \mu'^{I}_0\,(L_I\circ H_u) + \lambda''^{IJK}_3\,U_ID_JD_K$$

Superpotential (construction)



Useful formulae: $\mathcal{F}(\Phi)\big|_D \equiv \int \mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta\ \mathcal{F}(\Phi)$, $\mathcal{F}(\Phi)\big|_F \equiv \int \mathrm{d}^2\theta\ \mathcal{F}(\Phi)$,

In the WZ gauge: $\mathrm{e}^V = 1 + V + \frac{1}{2}V^2$, $V^2 = \frac{1}{2}\theta\theta\,\bar\theta\bar\theta\,V_\mu V^\mu$, $V^n = 0\ (n \geq 3)$.