The field content of the Standard Model (SM)

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The SM Lagrangian = Sum of the kinetic terms
+ the Higgs potential
+ the Yukawa interaction terms

Vector bosons		Field	Adj.repr.	Charge	Kinetic term	Covariant derivative	Field tensor	
$\mathrm{U(1)_Y}$	B (γ)	B_μ, ($B_\mu = B_\mu^\dagger$)	$\mathbf{1},\mathbf{1},0$	Weak hypercharge, Y	$-\frac14 B_{\mu\nu}B^{\mu\nu}$	$D_\mu = \partial_\mu + \dots + Y\,\mathrm{i}g'B_\mu$	$B_{\mu\nu} \equiv 2\partial_{[\mu}B_{\nu]}$	
$\mathrm{SU(2)_L}$	$W^\pm$, W^3 (Z^0)	W^i_μ, $i = 1,2,3$ ($W^i_\mu = W^{\dagger i}_\mu$)	$\mathbf{1},\mathbf{3},0$	Weak isospin, T^3	$-\frac14 W^i_{\mu\nu}W^{i\,\mu\nu} = -\frac12\,\mathrm{Tr}(\boldsymbol{W}_{\mu\nu}\boldsymbol{W}^{\mu\nu})$	$D_\mu = \partial_\mu + \dots + \mathrm{i}g\boldsymbol{W}_\mu$ $[D_\mu, D_\nu] = \dots + \mathrm{i}g\boldsymbol{W}_{\mu\nu}$	$\boldsymbol{W}_{\mu\nu} \equiv 2\partial_{[\mu}\boldsymbol{W}_{\nu]} + \mathrm{i}g[\boldsymbol{W}_\mu, \boldsymbol{W}_\nu]$ $W^i_{\mu\nu} = 2\partial_{[\mu}W^i_{\nu]} - g\epsilon^{ijk}W^j_\mu W^k_\nu$	$\boldsymbol{W}_\mu \equiv W^i_\mu t^i$ $\boldsymbol{W}_{\mu\nu} \equiv W^i_{\mu\nu}t^i$
$\mathrm{SU(3)_C}$	g	G^a_μ, $a = 1,\dots,8$ ($G^a_\mu = G^{\dagger a}_\mu$)	$\mathbf{8},\mathbf{1},0$	Color	$-\frac14 G^a_{\mu\nu}G^{a\,\mu\nu} = -\frac12\,\mathrm{Tr}(\boldsymbol{G}_{\mu\nu}\boldsymbol{G}^{\mu\nu})$	$D_\mu = \partial_\mu + \dots + \mathrm{i}g_s\boldsymbol{G}_\mu$ $[D_\mu, D_\nu] = \dots + \mathrm{i}g_s\boldsymbol{G}_{\mu\nu}$	$\boldsymbol{G}_{\mu\nu} = 2\partial_{[\mu}\boldsymbol{G}_{\nu]} + \mathrm{i}g_s[\boldsymbol{G}_\mu, \boldsymbol{G}_\nu]$ $G^a_{\mu\nu} = 2\partial_{[\mu}G^a_{\nu]} - g_s f^{abc}G^b_\mu G^c_\nu$	$\boldsymbol{G}_\mu \equiv G^a_\mu T^a$ $\boldsymbol{G}_{\mu\nu} \equiv G^a_{\mu\nu}T^a$

The $\mathrm{SU}(n)$ generators		
SU(2)	$t^i \equiv \frac12\tau^i$	$[t^i, t^j] = \mathrm{i}\epsilon^{ijk}t^k$ $\mathrm{Tr}(t^it^j) = \frac12\delta^{ij}$
SU(3)	$T^a \equiv \frac12\lambda^a$	$[T^a, T^b] = \mathrm{i}f^{abc}T^c$ $\mathrm{Tr}(T^aT^b) = \frac12\delta^{ab}$

Fermions		Field	Repr.	Q =	T^3 +	Y	Kinetic term	Covariant derivative
Quarks	$u_\mathrm{L}, (c_\mathrm{L}, t_\mathrm{L})$	$\boldsymbol{\Psi}_Q = \begin{pmatrix}\Psi^\mathrm{L}_u\\ \Psi^\mathrm{L}_d\end{pmatrix}$, $\boldsymbol{\Psi}_Q^\dagger$, $\Psi^\mathrm{L}_u = P_\mathrm{L}\Psi_u$, ...	$\mathbf{3},\mathbf{2},+\frac16$	+2/3	+1/2	+1/6	$\overline{\boldsymbol{\Psi}}_Q\ \mathrm{i}\gamma^\mu D_\mu\,\boldsymbol{\Psi}_Q$	$D_\mu\boldsymbol{\Psi}_Q = (\partial_\mu + \mathrm{i}g_s\boldsymbol{G}_\mu + \mathrm{i}g\boldsymbol{W}_\mu + \frac16\,\mathrm{i}g'B_\mu)\boldsymbol{\Psi}_Q$
	$d_\mathrm{L}, (s_\mathrm{L}, b_\mathrm{L})$			−1/3	−1/2	+1/6		
	$u_\mathrm{R}, (c_\mathrm{R}, t_\mathrm{R})$	$\Psi^\mathrm{R}_u = P_\mathrm{R}\Psi_u$, $\Psi^{\mathrm{R}\dagger}_u$	$\mathbf{3},\mathbf{1},+\frac23$	+2/3	0	+2/3	$\overline{\Psi}{}^\mathrm{R}_u\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_u$	$D_\mu\Psi^\mathrm{R}_u = (\partial_\mu + \mathrm{i}g_s\boldsymbol{G}_\mu + \boxed{\cancel{\mathrm{i}g\boldsymbol{W}_\mu}} + \frac23\,\mathrm{i}g'B_\mu)\Psi^\mathrm{R}_u$
	$d_\mathrm{R}, (s_\mathrm{R}, b_\mathrm{R})$	$\Psi^\mathrm{R}_d = P_\mathrm{R}\Psi_d$, $\Psi^{\mathrm{R}\dagger}_d$	$\mathbf{3},\mathbf{1},-\frac13$	−1/3	0	−1/3	$\overline{\Psi}{}^\mathrm{R}_d\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_d$	$D_\mu\Psi^\mathrm{R}_d = (\partial_\mu + \mathrm{i}g_s\boldsymbol{G}_\mu + \boxed{\cancel{\mathrm{i}g\boldsymbol{W}_\mu}} - \frac13\,\mathrm{i}g'B_\mu)\Psi^\mathrm{R}_d$
	$\bar{u}_\mathrm{L} = (u_\mathrm{R})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar u} = P_\mathrm{L}\Psi_{\bar u}$, $\Psi^{\mathrm{L}\dagger}_{\bar u}$	$\bar{\mathbf{3}},\mathbf{1},-\frac23$	−2/3	0	−2/3	$\overline{\Psi}{}^\mathrm{L}_{\bar u}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar u}$	$D_\mu\Psi^\mathrm{L}_{\bar u} = (\partial_\mu - \mathrm{i}g_s\boldsymbol{G}^*_\mu - \boxed{\cancel{\mathrm{i}g\boldsymbol{W}^*_\mu}} + \frac23\,\mathrm{i}g'B_\mu)\Psi^\mathrm{L}_{\bar u}$
	$\bar{d}_\mathrm{L} = (d_\mathrm{R})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar d} = P_\mathrm{L}\Psi_{\bar d}$, $\Psi^{\mathrm{L}\dagger}_{\bar d}$	$\bar{\mathbf{3}},\mathbf{1},+\frac13$	+1/3	0	+1/3	$\overline{\Psi}{}^\mathrm{L}_{\bar d}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar d}$	$D_\mu\Psi^\mathrm{L}_{\bar d} = (\partial_\mu - \mathrm{i}g_s\boldsymbol{G}^*_\mu - \boxed{\cancel{\mathrm{i}g\boldsymbol{W}^*_\mu}} - \frac13\,\mathrm{i}g'B_\mu)\Psi^\mathrm{L}_{\bar d}$
Leptons	$\nu_{e\mathrm{L}}, (\nu_{\mu\mathrm{L}}, \nu_{\tau\mathrm{L}})$	$\boldsymbol{\Psi}_L = \begin{pmatrix}\Psi^\mathrm{L}_{\nu_e}\\ \Psi^\mathrm{L}_e\end{pmatrix}$, $\boldsymbol{\Psi}_L^\dagger$, $\Psi^\mathrm{L}_{\nu_e} = P_\mathrm{L}\Psi_{\nu_e}$, ...	$\mathbf{1},\mathbf{2},-\frac12$	0	+1/2	−1/2	$\overline{\boldsymbol{\Psi}}_L\ \mathrm{i}\gamma^\mu D_\mu\,\boldsymbol{\Psi}_L$	$D_\mu\boldsymbol{\Psi}_L = (\partial_\mu + \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}_\mu}} + \mathrm{i}g\boldsymbol{W}_\mu - \frac12\,\mathrm{i}g'B_\mu)\boldsymbol{\Psi}_L$
	$e_\mathrm{L}, (\mu_\mathrm{L}, \tau_\mathrm{L})$			−1	−1/2	−1/2		
	$\nu_{e\mathrm{R}}, (\nu_{\mu\mathrm{R}}, \nu_{\tau\mathrm{R}})$	$\Psi^\mathrm{R}_{\nu_e} = P_\mathrm{R}\Psi_{\nu_e}$, $\Psi^{\mathrm{R}\dagger}_{\nu_e}$	$\mathbf{1},\mathbf{1},0$	0	0	0	$\overline{\Psi}{}^\mathrm{R}_{\nu_e}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_{\nu_e}$	$D_\mu\Psi^\mathrm{L}_{\nu_e} = (\partial_\mu + \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}_\mu + \mathrm{i}g\boldsymbol{W}_\mu + \mathrm{i}g'Y B_\mu}})\Psi^\mathrm{L}_{\nu_e}$
	$e_\mathrm{R}, (\mu_\mathrm{R}, \tau_\mathrm{R})$	$\Psi^\mathrm{R}_e = P_\mathrm{R}\Psi_e$, $\Psi^{\mathrm{R}\dagger}_e$	$\mathbf{1},\mathbf{1},-1$	−1	0	−1	$\overline{\Psi}{}^\mathrm{R}_e\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{R}_e$	$D_\mu\Psi^\mathrm{R}_e = (\partial_\mu + \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}_\mu + \mathrm{i}g\boldsymbol{W}_\mu}} - \mathrm{i}g'B_\mu)\Psi^\mathrm{R}_e$
	$\bar\nu_{e\mathrm{L}} = (\nu_{e\mathrm{R}})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar\nu_e} = P_\mathrm{L}\Psi_{\bar\nu_e}$, $\Psi^{\mathrm{L}\dagger}_{\bar\nu_e}$	$\mathbf{1},\mathbf{1},0$	0	0	0	$\overline{\Psi}{}^\mathrm{L}_{\bar\nu_e}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar\nu_e}$	$D_\mu\Psi^\mathrm{L}_{\bar\nu_e} = (\partial_\mu - \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}^*_\mu - \mathrm{i}g\boldsymbol{W}^*_\mu + \mathrm{i}g'Y B_\mu}})\Psi^\mathrm{L}_{\bar\nu_e}$
	$\bar{e}_\mathrm{L} = (e_\mathrm{R})^\mathrm{c}$, ...	$\Psi^\mathrm{L}_{\bar e} = P_\mathrm{L}\Psi_{\bar e}$, $\Psi^{\mathrm{L}\dagger}_{\bar e}$	$\mathbf{1},\mathbf{1},+1$	+1	0	+1	$\overline{\Psi}{}^\mathrm{L}_{\bar e}\ \mathrm{i}\gamma^\mu D_\mu\,\Psi^\mathrm{L}_{\bar e}$	$D_\mu\Psi^\mathrm{L}_{\bar e} = (\partial_\mu - \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}^*_\mu - \mathrm{i}g\boldsymbol{W}^*_\mu}} + \mathrm{i}g'B_\mu)\Psi^\mathrm{L}_{\bar e}$

Scalar boson		Field	Repr.	Q	T^3	Y	Kinetic term	Covariant derivative
Higgs	H^+, H^-	$\boldsymbol{\Phi} = \begin{pmatrix}H^+\\ H^0\end{pmatrix}$, $\boldsymbol{\Phi}^\dagger$, $\boldsymbol{\Phi}^\dagger = (H^-\ \ H^{0\dagger})$	$\mathbf{1},\mathbf{2},+\frac12$	+1	+1/2	+1/2	$(D_\mu\boldsymbol{\Phi})^\dagger D^\mu\boldsymbol{\Phi}$	$D_\mu\boldsymbol{\Phi} = (\partial_\mu + \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}_\mu}} + \mathrm{i}g\boldsymbol{W}_\mu + \frac12\,\mathrm{i}g'B_\mu)\boldsymbol{\Phi}$ $(D_\mu\boldsymbol{\Phi})^* = (\partial_\mu - \boxed{\cancel{\mathrm{i}g_s\boldsymbol{G}^*_\mu}} - \mathrm{i}g\boldsymbol{W}^*_\mu - \frac12\,\mathrm{i}g'B_\mu)\boldsymbol{\Phi}^*$
	H^0, $H^{0\dagger}$			0	−1/2	+1/2		

The representation of $\widetilde{\boldsymbol{\Phi}}$ is $(\mathbf{1}, \bar{\mathbf{2}}, -\frac12)$.

Chirality projection: $\Psi \equiv \begin{pmatrix}\chi_\alpha\\ \bar\psi^{\dot\alpha}\end{pmatrix}$, $\boxed{\Psi^\mathrm{L} \equiv P_\mathrm{L}\Psi} = \begin{pmatrix}\chi_\alpha\\ 0\end{pmatrix}$, $\boxed{\Psi^\mathrm{R} \equiv P_\mathrm{R}\Psi} = \begin{pmatrix}0\\ \bar\psi^{\dot\alpha}\end{pmatrix}$

Dirac conjugate: $\boxed{\overline{\Psi} \equiv \Psi^\dagger\beta}$, $\beta \equiv \begin{pmatrix}0 & \delta^\alpha{}_\beta\\ \delta_{\dot\alpha}{}^{\dot\beta} & 0\end{pmatrix}$, $\Psi^\dagger = (\bar\chi^{\dot\alpha}\ \ \psi_\alpha)$, $\overline{\Psi} = (\psi_\alpha\ \ \bar\chi^{\dot\alpha})$

Charge conjugate: $\boxed{\Psi^\mathrm{c} \equiv C_0\Psi^*}$, $C_0 \equiv \begin{pmatrix}0 & \varepsilon_{\alpha\beta}\\ \varepsilon^{\dot\alpha\dot\beta} & 0\end{pmatrix}$, $\Psi^\mathrm{c} = \begin{pmatrix}\psi_\alpha\\ \bar\chi^{\dot\alpha}\end{pmatrix}$

$$(D_\mu\Psi)^\mathrm{c} = C_0(D_\mu\Psi)^* = D^*_\mu\Psi^\mathrm{c}$$

$^\mathsf{T}$ transpose, * complex conjugate, † hermitian conjugate
$\alpha, \beta, \dots$ L-type 2-component spinor indices (dotted = R-type)

Trading R-chiral for the charge conjugated L-chiral fields:

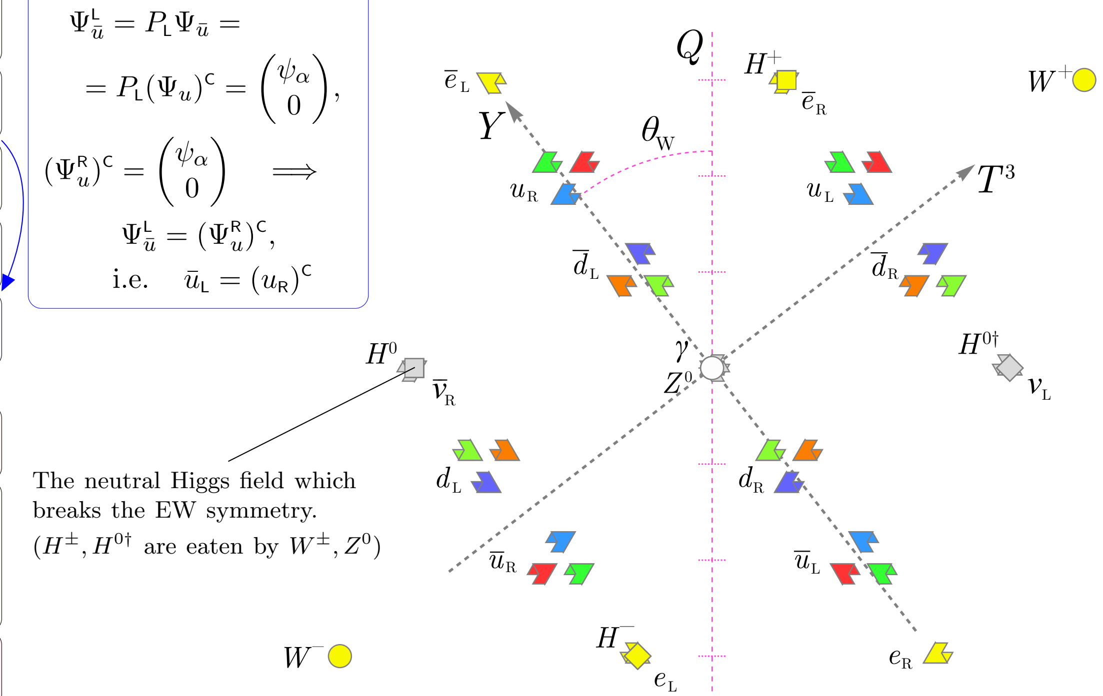
$$\Psi_{\bar u} = (\Psi_u)^\mathrm{c} = \begin{pmatrix}\psi_\alpha\\ \bar\chi^{\dot\alpha}\end{pmatrix},$$

$$\Psi^\mathrm{L}_{\bar u} = P_\mathrm{L}\Psi_{\bar u} = P_\mathrm{L}(\Psi_u)^\mathrm{c} = \begin{pmatrix}\psi_\alpha\\ 0\end{pmatrix},$$

$$(\Psi^\mathrm{R}_u)^\mathrm{c} = \begin{pmatrix}\psi_\alpha\\ 0\end{pmatrix} \implies \Psi^\mathrm{L}_{\bar u} = (\Psi^\mathrm{R}_u)^\mathrm{c},$$

i.e. $\bar u_\mathrm{L} = (u_\mathrm{R})^\mathrm{c}$

The electroweak charges



The interaction terms

The interactions between the gauge bosons and the other fields (fermions and Higgs) all arise from the gauge covariant derivatives. The self-interactions of the nonabelian gauge bosons are all contained in their kinetic terms. The interactions between the fermions and the Higgs field are all given by Yukawa terms (on SSB, these terms generate the fermion masses).

The Yukawa interaction terms

$$-\left\{ y_d^{IJ}\left(\overline{\boldsymbol{\Psi}}{}^\mathrm{L}_{Q_I}\boldsymbol{\Phi}\right)\Psi^\mathrm{R}_{d_J} + y_u^{IJ}\left(\overline{\boldsymbol{\Psi}}{}^\mathrm{L}_{Q_I}\widetilde{\boldsymbol{\Phi}}\right)\Psi^\mathrm{R}_{u_J} + y_\ell^{IJ}\left(\overline{\boldsymbol{\Psi}}{}^\mathrm{L}_{L_I}\boldsymbol{\Phi}\right)\Psi^\mathrm{R}_{\ell_J} + y_\nu^{IJ}\left(\overline{\boldsymbol{\Psi}}{}^\mathrm{L}_{L_I}\widetilde{\boldsymbol{\Phi}}\right)\Psi^\mathrm{R}_{\nu_J} + \text{h.c.}\right\},$$

where $\widetilde{\boldsymbol{\Phi}} \equiv (-\mathrm{i}\tau^2)^\mathsf{T}\boldsymbol{\Phi}^*$, (Note: $\boldsymbol{\Phi}_d = \boldsymbol{\Phi}$, $\boldsymbol{\Phi}_u = \widetilde{\boldsymbol{\Phi}}$)
and the indices $I, J = 1,2,3$ run over generations

The Higgs potential

$$\mathcal{L}^\mathrm{H} = -\mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

For **SSB**: $\mu^2 < 0$ and $\lambda > 0$

vev: $\left\langle\boldsymbol{\Phi}^\dagger\boldsymbol{\Phi}\right\rangle_\mathrm{min} = \dfrac{v^2}{2}$, $v = \sqrt{\dfrac{-\mu^2}{\lambda}} > 0$,

unitary g.: $\boldsymbol{\Phi} = \frac{1}{\sqrt2}\begin{pmatrix}0\\ v + \sigma(x)\end{pmatrix}$, $\boldsymbol{\Phi}_0 = \frac{1}{\sqrt2}\begin{pmatrix}0\\ v\end{pmatrix}$

$$\begin{pmatrix}W^+_\mu\\ W^-_\mu\end{pmatrix} = \frac{1}{\sqrt2}\begin{pmatrix}1 & -\mathrm{i}\\ \mathrm{i} & 1\end{pmatrix}\begin{pmatrix}W^1_\mu\\ W^2_\mu\end{pmatrix}, \quad \begin{pmatrix}Z_\mu\\ A_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta_\mathrm{W} & -\sin\theta_\mathrm{W}\\ \sin\theta_\mathrm{W} & \cos\theta_\mathrm{W}\end{pmatrix}\begin{pmatrix}W^3_\mu\\ B_\mu\end{pmatrix}$$

The Weinberg angle: $\tan\theta_\mathrm{W} \equiv \dfrac{g'}{g}$, $e = g'\cos\theta_\mathrm{W} = g\sin\theta_\mathrm{W}$,

$$\sin\theta_\mathrm{W} \equiv \frac{g'}{\sqrt{g^2 + g'^2}}, \quad \cos\theta_\mathrm{W} \equiv \frac{g}{\sqrt{g^2 + g'^2}}$$