Tutorial 14

FK8027 - Quantum Field Theory

Monday 19$^{\text{th}}$ February, 2018

Topics for today

- Problems 7.1, 7.3, 7.5 in M&S
- Feynman rules for QED

1 Bhabha scattering

Problem 7.1. Derive the lowest-order non-vanishing S-matrix element (7.19) and hence the corresponding Feynman amplitude for Bhabha scattering, i.e., the process

$$e^-\left(\vec{p_1}, r_1\right) + e^+\left(\vec{p_2}, r_2\right) \to e^-\left(\vec{p_1'}, s_1\right) + e^+\left(\vec{p_2'}, s_2\right).$$

Solution. Bhabha scattering is the elastic process $e^-e^+ \to e^-e^+$ [M&S, pp. 107, 139]. Even intuitively, e^- and e^+ can interact in two ways: they can exchange some energy and momentum (the proper scattering) or they can annihilate producing a photon, which then produces another e^-e^+ pair.

Since there cannot be first-order processes in QED [M&S, Sec. 7.2.1], we consider the second-order contribution to the S-matrix, i.e.,

$$S^{(2)} = \frac{(-\mathrm{i})^2}{2!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\ \mathrm{T}\left[\mathcal{H}_\mathrm{I}(x_1)\mathcal{H}_\mathrm{I}(x_2)\right], \tag{1}$$

where $\mathrm{T}[\cdot]$ is the time-ordered product and $\mathcal{H}_\mathrm{I}(x) = -e\,\mathrm{N}\left[\overline{\psi}(x)\not{A}(x)\psi(x)\right]$, with $\mathrm{N}[\cdot]$ normal ordering operator and e charge of the electron. The expansion of the time-ordered product by means of the Wick's theorem [M&S, Sec. 6.3], gives all possible second-order processes. We only need the term related to Bhabha scattering. Following [M&S, Chap. 6, p.107], we note that there are no photons in the initial and final states, and therefore the term giving rise to Bhabha scattering must be the one with the contraction between the two electromagnetic potentials $A_\mu(x_1)$ and $A_\nu(x_2)$,

$$\begin{aligned} S^{(2)}_{\text{Bhabha}} =& \frac{(-\mathrm{i}e)^2}{2!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\ \mathrm{N}\left[\overline{\psi}(x_1)\gamma^\mu\psi(x_1)\overline{\psi}(x_2)\gamma^\nu\psi(x_2)\right]\cdot \\ &\qquad\cdot\langle 0|\,\mathrm{T}\left[A_\mu(x_1)A_\nu(x_2)\right]|0\rangle \\ =& \frac{(-\mathrm{i}e)^2}{2!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\ \mathrm{N}\left[\overline{\psi}(x_1)\gamma^\mu\psi(x_1)\overline{\psi}(x_2)\gamma^\nu\psi(x_2)\right]\mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(x_1-x_2). \end{aligned} \tag{2}$$

In other words, the electromagnetic potential cannot appear uncontracted, because it would annihilate photons in the initial states or create photons in the final state, but we are not interested in these kind of processes in this exercise. In (2), we have

$$\begin{aligned}
\mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(x_1 - x_2) &= \frac{\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k\, \mathcal{D}^{\mathrm{F}}_{\mu\nu}(k)\mathrm{e}^{-\mathrm{i}k(x_1-x_2)} \qquad (3a)\\
&= \frac{\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k\, \frac{-\eta_{\mu\nu}}{k^2+\mathrm{i}\epsilon}\mathrm{e}^{-\mathrm{i}k(x_1-x_2)},
\end{aligned}$$

$$\begin{aligned}
\psi_t(x) &= \psi_t^+(x) + \psi_t^-(x)\\
&= \sum_{t=1}^{2}\int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{k}}}}\left(c_t\left(\vec{k}\right)u_t\left(\vec{k}\right)\mathrm{e}^{-\mathrm{i}kx} + d_t^\dagger\left(\vec{k}\right)v_t\left(\vec{k}\right)\mathrm{e}^{\mathrm{i}kx}\right), \qquad (3b)
\end{aligned}$$

$$\begin{aligned}
\overline{\psi}_t(x) &= \overline{\psi}_t^-(x) + \overline{\psi}_t^+(x)\\
&= \sum_{t=1}^{2}\int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{k}}}}\left(c_t^\dagger\left(\vec{k}\right)\overline{u}_t\left(\vec{k}\right)\mathrm{e}^{\mathrm{i}kx} + d_t\left(\vec{k}\right)\overline{v}_t\left(\vec{k}\right)\mathrm{e}^{-\mathrm{i}kx}\right). \qquad (3c)
\end{aligned}$$

In $S^{(2)}_{\mathrm{Bhabha}}$, the normal product contains 16 terms coming from the products between the fermionic fields. When we evaluate $\langle f|\, S^{(2)}_{\mathrm{Bhabha}}\,|i\rangle$, with $|i\rangle = c^\dagger_{r_1}\left(\vec{p_1}\right)d^\dagger_{r_2}\left(\vec{p_2}\right)|0\rangle$ and $|f\rangle = c^\dagger_{s_1}\left(\vec{p_1'}\right)d^\dagger_{s_2}\left(\vec{p_2'}\right)|0\rangle$, only 4 of these terms will contribute to the S-matrix element. These are the ones containing the correct annihilation operators to annihilate the initial state and the correct creation operators to create the final state [M&S, Chap. 6]. The other 12 terms give rise to scalar products between orthogonal states, which are zero, hence there is no need to consider them. You can verify this explicitly by expanding the terms in the normal product in full generality, then normal ordering the fermionic operators and, finally, commuting and anticommuting the operators to have all the annihilation operators to the right and all the creation operators to the left. We are going to follow this procedure in full detail for the 4 non-zero terms only. The normal product under consideration is,

$$\begin{aligned}
\mathrm{N}\left[\overline{\psi}(x_1)\gamma^\mu\psi(x_1)\overline{\psi}(x_2)\gamma^\nu\psi(x_2)\right] = \mathrm{N}\Big[&\left(\overline{\psi}_t^+(x_1) + \overline{\psi}_t^-(x_1)\right)\gamma^\mu\left(\psi_t^+(x_1) + \psi_t^-(x_1)\right)\cdot\\
&\cdot\left(\overline{\psi}_t^+(x_2) + \overline{\psi}_t^-(x_2)\right)\gamma^\nu\left(\psi_t^+(x_2) + \psi_t^-(x_2)\right)\Big], \qquad (4)
\end{aligned}$$

and the non-zero terms are

$$\mathrm{N}\left[\overline{\psi}_t^-(x_1)\gamma^\mu\psi_t^+(x_1)\overline{\psi}_t^+(x_2)\gamma^\nu\psi_t^-(x_2)\right] + \mathrm{N}\left[\overline{\psi}_t^+(x_1)\gamma^\mu\psi_t^-(x_1)\overline{\psi}_t^-(x_2)\gamma^\nu\psi_t^+(x_2)\right]$$

$$+ \mathrm{N}\left[\overline{\psi}_t^-(x_1)\gamma^\mu\psi_t^-(x_1)\overline{\psi}_t^+(x_2)\gamma^\nu\psi_t^+(x_2)\right] + \mathrm{N}\left[\overline{\psi}_t^+(x_1)\gamma^\mu\psi_t^+(x_1)\overline{\psi}_t^-(x_2)\gamma^\nu\psi_t^-(x_2)\right]. \tag{5}$$

The first two terms in (5) are equivalent, as the last two terms. In fact, the variables x_1, x_2 are dummy variables since they are integrated in the S-matrix expansion, therefore we can interchange them; in addition, $\mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}$ is clearly symmetric in μ, ν since the metric is symmetric, therefore we can also interchange μ and ν. Hence, we end up with the following expression,

$$\begin{aligned}\langle f|\, S^{(2)}_{\mathrm{Bhabha}}\,|i\rangle = \frac{(-\mathrm{i}e)^2}{2!}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\,\mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(x_1-x_2)\cdot\\ \cdot\, 2\langle f|\Big\{\mathrm{N}\left[\overline{\psi}_t^-(x_1)\gamma^\mu\psi_t^+(x_1)\overline{\psi}_t^+(x_2)\gamma^\nu\psi_t^-(x_2)\right]\\ + \mathrm{N}\left[\overline{\psi}_t^-(x_1)\gamma^\mu\psi_t^-(x_1)\overline{\psi}_t^+(x_2)\gamma^\nu\psi_t^+(x_2)\right]\Big\}\,|i\rangle.\end{aligned} \tag{6}$$

We repeat again that these two non-zero terms *exactly contain the creation and annihilation operators needed to annihilate the particles in $|i\rangle$ and create the particles in $|f\rangle$*.

Now we plug the explicit expressions of the fields in the matrix element,

$$\begin{aligned}\langle f|\, S^{(2)}_{\mathrm{Bhabha}}\,|i\rangle = (-\mathrm{i}e)^2\int \mathrm{d}^4x_1\mathrm{d}^4x_2\,\mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(x_1-x_2)\cdot\\ \cdot\Bigg\{\langle f|\,\mathrm{N}\Bigg[\sum_{t=1}^{2}\int\frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{k}}}}\left(c_t^\dagger(\vec{k})\,\overline{u}_t(\vec{k})\,\mathrm{e}^{\mathrm{i}kx_1}\right)\gamma^\mu\\ \sum_{q=1}^{2}\int\frac{\mathrm{d}^3k'}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{k}'}}}\left(c_q(\vec{k}')\,u_q(\vec{k}')\,\mathrm{e}^{-\mathrm{i}k'x_1}\right)\\ \sum_{t'=1}^{2}\int\frac{\mathrm{d}^3\lambda}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{\lambda}}}}\left(d_{t'}(\vec{\lambda})\,\overline{v}_{t'}(\vec{\lambda})\,\mathrm{e}^{-\mathrm{i}\lambda x_2}\right)\gamma^\nu\\ \sum_{q'=1}^{2}\int\frac{\mathrm{d}^3\lambda'}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{\lambda}'}}}\left(d_{q'}^\dagger(\vec{\lambda}')\,v_{q'}(\vec{\lambda}')\,\mathrm{e}^{\mathrm{i}\lambda'x_2}\right)\Bigg]|i\rangle\\ +\langle f|\,\mathrm{N}\Bigg[\sum_{t=1}^{2}\int\frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{k}}}}\left(c_t^\dagger(\vec{k})\,\overline{u}_t(\vec{k})\,\mathrm{e}^{\mathrm{i}kx_1}\right)\gamma^\mu\\ \sum_{q=1}^{2}\int\frac{\mathrm{d}^3k'}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{k}'}}}\left(d_q^\dagger(\vec{k}')\,v_q(\vec{k}')\,\mathrm{e}^{\mathrm{i}k'x_1}\right)\\ \sum_{t'=1}^{2}\int\frac{\mathrm{d}^3\lambda}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{\lambda}}}}\left(d_{t'}(\vec{\lambda})\,\overline{v}_{t'}(\vec{\lambda})\,\mathrm{e}^{-\mathrm{i}\lambda x_2}\right)\gamma^\nu\\ \sum_{q'=1}^{2}\int\frac{\mathrm{d}^3\lambda'}{(2\pi)^{3/2}}\sqrt{\frac{m}{E_{\vec{\lambda}'}}}\left(c_{q'}(\vec{\lambda}')\,u_{q'}(\vec{\lambda}')\,\mathrm{e}^{-\mathrm{i}\lambda'x_2}\right)\Bigg]|i\rangle\Bigg\}.\end{aligned} \tag{7}$$

This expression can be rewritten as,

$$\langle f| S^{(2)}_{\mathrm{Bhabha}} |i\rangle = (-\mathrm{i}e)^2 \int \mathrm{d}^4x_1 \mathrm{d}^4x_2\, \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(x_1 - x_2)\cdot$$
$$\cdot \sum_{t=1}^{2}\sum_{q=1}^{2}\sum_{t'=1}^{2}\sum_{q'=1}^{2} \int \frac{\mathrm{d}^4k\mathrm{d}^4k'\mathrm{d}^4\lambda\mathrm{d}^4\lambda'}{(2\pi)^6} \frac{m^2}{\sqrt{E_{\vec{k}}E_{\vec{k'}}E_{\vec{\lambda}}E_{\vec{\lambda'}}}}\cdot$$
$$\cdot \Big\{ \overline{u}_t(\vec{k})\,\gamma^\mu u_q(\vec{k'})\,\mathrm{e}^{-\mathrm{i}(k'-k)x_1}\overline{v}_{t'}(\vec{\lambda})\,\gamma^\nu v_{q'}(\vec{\lambda'})\,\mathrm{e}^{\mathrm{i}(\lambda'-\lambda)x_2}\cdot$$
$$\cdot \boxed{\langle 0| c_{s_1}(\vec{p_1'})\, d_{s_2}(\vec{p_2'})\, \mathrm{N}\left[c_t^\dagger(\vec{k})\, c_q(\vec{k'})\, d_{t'}(\vec{\lambda})\, d^\dagger_{q'}(\vec{\lambda'})\right] c^\dagger_{r_1}(\vec{p_1})\, d^\dagger_{r_2}(\vec{p_2})\,|0\rangle}$$
$$+ \overline{u}_t(\vec{k})\,\gamma^\mu v_q(\vec{k'})\,\mathrm{e}^{\mathrm{i}(k'+k)x_1}\overline{v}_{t'}(\vec{\lambda})\,\gamma^\nu u_{q'}(\vec{\lambda'})\,\mathrm{e}^{-\mathrm{i}(\lambda'+\lambda)x_2}\cdot$$
$$\cdot \boxed{\langle 0| c_{s_1}(\vec{p_1'})\, d_{s_2}(\vec{p_2'})\, \mathrm{N}\left[c_t^\dagger(\vec{k})\, d^\dagger_q(\vec{k'})\, d_{t'}(\vec{\lambda})\, c_{q'}(\vec{\lambda'})\right] c^\dagger_{r_1}(\vec{p_1})\, d^\dagger_{r_2}(\vec{p_2})\,|0\rangle}\Big\}. \tag{8}$$

Let's consider the vacuum expectation values in the boxes; the first is

$$\langle 0| c_{s_1}(\vec{p_1'})\, d_{s_2}(\vec{p_2'})\, \mathrm{N}\left[c_t^\dagger(\vec{k})\, c_q(\vec{k'})\, d_{t'}(\vec{\lambda})\, d^\dagger_{q'}(\vec{\lambda'})\right] c^\dagger_{r_1}(\vec{p_1})\, d^\dagger_{r_2}(\vec{p_2})\,|0\rangle$$
$$= -\langle 0| c_{s_1}(\vec{p_1'})\, d_{s_2}(\vec{p_2'})\, c_t^\dagger(\vec{k})\, c_q(\vec{k'})\, d^\dagger_{q'}(\vec{\lambda'})\, d_{t'}(\vec{\lambda})\, c^\dagger_{r_1}(\vec{p_1})\, d^\dagger_{r_2}(\vec{p_2})\,|0\rangle, \tag{9}$$

where the minus sign comes from the anticommutation of $d_{t'}(\vec{\lambda})$ and $d^\dagger_{q'}(\vec{\lambda'})$ when applying the normal ordering. We remind the only non-trivial anti-commutation rules,

$$\left\{c_r(\vec{p}), c_s^\dagger(\vec{p'})\right\} = \left\{d_r(\vec{p}), d_s^\dagger(\vec{p'})\right\} = \delta_{rs}\delta_{\vec{p}\vec{p'}}. \tag{10}$$

At this point, we want to write all the annihilation operators to the right and all the creation operators to the left by commuting and anticommuting them. So we commute and anticommute the operators in the boxes in (11), where the subscripts C and A tell us that the operators inside the boxes commute or anticommute, respectively,

$$-\langle 0| c_{s_1}(\vec{p_1'})\, \boxed{d_{s_2}(\vec{p_2'})\, c_t^\dagger(\vec{k})}_{\mathrm{C}}\, \boxed{c_q(\vec{k'})\, d^\dagger_{q'}(\vec{\lambda'})}_{\mathrm{C}}\, \boxed{d_{t'}(\vec{\lambda})\, c^\dagger_{r_1}(\vec{p_1})}_{\mathrm{C}}\, d^\dagger_{r_2}(\vec{p_2})\,|0\rangle$$
$$= -\langle 0| c_{s_1}(\vec{p_1'})\, c_t^\dagger(\vec{k})\, d_{s_2}(\vec{p_2'})\, d^\dagger_{q'}(\vec{\lambda'})\, c_q(\vec{k'})\, c^\dagger_{r_1}(\vec{p_1})\, d_{t'}(\vec{\lambda})\, d^\dagger_{r_2}(\vec{p_2})\,|0\rangle$$
$$-\langle 0| \boxed{c_{s_1}(\vec{p_1'})\, c_t^\dagger(\vec{k})}_{\mathrm{A}}\, \boxed{d_{s_2}(\vec{p_2'})\, d^\dagger_{q'}(\vec{\lambda'})}_{\mathrm{A}}\, \boxed{c_q(\vec{k'})\, c^\dagger_{r_1}(\vec{p_1})}_{\mathrm{A}}\, \boxed{d_{t'}(\vec{\lambda})\, d^\dagger_{r_2}(\vec{p_2})}_{\mathrm{A}}\,|0\rangle$$
$$= -\langle 0| \left(-c_t^\dagger(\vec{k})\, c_{s_1}(\vec{p_1'}) + \delta_{ts_1}\delta_{\vec{k}\vec{p_1'}}\right)\left(-d^\dagger_{t'}(\vec{\lambda})\, d_{s_2}(\vec{p_2'}) + \delta_{s_2t'}\delta_{\vec{\lambda}\vec{p_2'}}\right)\cdot$$
$$\cdot\left(-c^\dagger_{r_1}(\vec{p_1})\, c_q(\vec{k'}) + \delta_{r_1q}\delta_{\vec{k'}\vec{p_1}}\right)\left(-d^\dagger_{r_2}(\vec{p_2})\, d_{q'}(\vec{\lambda'}) + \delta_{r_2q'}\delta_{\vec{\lambda'}\vec{p_2}}\right)|0\rangle, \tag{11}$$

The only non-zero term here is the one containing all the deltas, because all the others have an annihilation operator acting on the vacuum state. Hence the result is,

$$-\langle 0|0\rangle\, \delta_{ts_1}\delta_{\vec{k}\vec{p_1'}}\, \delta_{s_2t'}\delta_{\vec{\lambda}\vec{p_2'}}\, \delta_{r_1q}\delta_{\vec{k'}\vec{p_1}}\, \delta_{r_2q'}\delta_{\vec{\lambda'}\vec{p_2}}, \tag{12}$$

with $\langle 0|0\rangle = 1$. The second vacuum expectation value is computed in the same way,

$$\begin{aligned}
&\langle 0|\, c_{s_1}(\vec{p_1'})\, d_{s_2}(\vec{p_2'})\, \mathrm{N}\left[c_t^\dagger(\vec{k})\, d_q^\dagger(\vec{k'})\, d_{t'}(\vec{\lambda})\, c_{q'}(\vec{\lambda'})\right] c_{r_1}^\dagger(\vec{p_1})\, d_{r_2}^\dagger(\vec{p_2})\,|0\rangle \\
&= \langle 0|\, c_{s_1}(\vec{p_1'}) \boxed{d_{s_2}(\vec{p_2'})\, c_t^\dagger(\vec{k})}_{\mathrm{C}} d_q^\dagger(\vec{k'}) \boxed{d_{t'}(\vec{\lambda})\, c_{q'}(\vec{\lambda'})}_{\mathrm{C}} c_{r_1}^\dagger(\vec{p_1})\, d_{r_2}^\dagger(\vec{p_2})\,|0\rangle \\
&= \langle 0|\, c_{s_1}(\vec{p_1'})\, c_t^\dagger(\vec{k})\, d_{s_2}(\vec{p_2'})\, d_q^\dagger(\vec{k'})\, c_{q'}(\vec{\lambda'}) \boxed{d_{t'}(\vec{\lambda})\, c_{r_1}^\dagger(\vec{p_1})}_{\mathrm{C}} d_{r_2}^\dagger(\vec{p_2})\,|0\rangle \\
&= \langle 0| \boxed{c_{s_1}(\vec{p_1'})\, c_t^\dagger(\vec{k})}_{\mathrm{A}} \boxed{d_{s_2}(\vec{p_2'})\, d_q^\dagger(\vec{k'})}_{\mathrm{A}} \boxed{c_{q'}(\vec{\lambda'})\, c_{r_1}^\dagger(\vec{p_1})}_{\mathrm{A}} \boxed{d_{t'}(\vec{\lambda})\, d_{r_2}^\dagger(\vec{p_2})}_{\mathrm{A}} |0\rangle \\
&= \langle 0| \left(-c_t^\dagger(\vec{k})\, c_{s_1}(\vec{p_1'}) + \delta_{ts_1}\delta_{\vec{k}\vec{p_1'}}\right)\left(-d_q^\dagger(\vec{k'})\, d_{s_2}(\vec{p_2'}) + \delta_{s_2q}\delta_{\vec{k'}\vec{p_2'}}\right)\cdot \\
&\qquad \cdot\left(-d_{r_2}^\dagger(\vec{p_2})\, d_{t'}(\vec{\lambda}) + \delta_{r_2t'}\delta_{\vec{\lambda}\vec{p_2}}\right)\left(-c_{r_1}^\dagger(\vec{p_1})\, c_{q'}(\vec{\lambda'}) + \delta_{r_1q'}\delta_{\vec{\lambda'}\vec{p_1}}\right)|0\rangle \\
&= \delta_{ts_1}\delta_{\vec{k}\vec{p_1'}}\, \delta_{s_2q}\delta_{\vec{k'}\vec{p_2'}}\, \delta_{r_2t'}\delta_{\vec{p_2}\vec{\lambda}}\, \delta_{r_1q'}\delta_{\vec{\lambda'}\vec{p_1}}.
\end{aligned} \tag{13}$$

Now we can use these deltas to remove the integrals over the four-momenta and the sums over the spins. Upon doing that one obtains,

$$\begin{aligned}
\langle f|\, S^{(2)}_{\mathrm{Bhabha}}\,|i\rangle = \frac{(-\mathrm{i}e)^2}{(2\pi)^6}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\, \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(x_1-x_2)\frac{m^2}{\sqrt{E_{\vec{p_1}}E_{\vec{p_2}}E_{\vec{p_1'}}E_{\vec{p_2'}}}}\cdot \\
\cdot\Big\{-\overline{u}_{s_1}(\vec{p_1'})\,\gamma^\mu u_r(\vec{p_1})\,\mathrm{e}^{\mathrm{i}\left(p_1'-p_1\right)x_1}\overline{v}_{s_2}(\vec{p_2'})\,\gamma^\nu v_{r_2}(\vec{p_2})\,\mathrm{e}^{-\mathrm{i}\left(p_2'-p_2\right)x_2} \\
+\,\overline{u}_{s_1}(\vec{p_1'})\,\gamma^\mu v_{s_2}(\vec{p_2'})\,\mathrm{e}^{\mathrm{i}\left(p_2'+p_1'\right)x_1}\overline{v}_{r_2}(\vec{p_2})\,\gamma^\nu u_{r_1}(\vec{p_1})\,\mathrm{e}^{-\mathrm{i}(p_2+p_1)x_2}\Big\}.
\end{aligned} \tag{14}$$

The next step is the computation of the integrals over x_1 and x_2, so we collect these two integrals explicitly and write down the expression for the Fourier transform of the photon propagator,

$$\begin{aligned}
\langle f|\, S^{(2)}_{\mathrm{Bhabha}}\,|i\rangle = \frac{(-\mathrm{i}e)^2}{(2\pi)^6}\frac{m^2}{\sqrt{E_{\vec{p_1}}E_{\vec{p_2}}E_{\vec{p_1'}}E_{\vec{p_2'}}}}\cdot \\
\cdot\Big\{-\overline{u}_{s_1}(\vec{p_1'})\,\gamma^\mu u_r(\vec{p_1})\,\overline{v}_{s_2}(\vec{p_2'})\,\gamma^\nu v_{r_2}(\vec{p_2})\cdot \\
\cdot\int\frac{\mathrm{d}^4q}{(2\pi)^4}\mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(q)\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{e}^{-\mathrm{i}q(x_1-x_2)}\mathrm{e}^{\mathrm{i}\left(p_1'-p_1\right)x_1}\mathrm{e}^{-\mathrm{i}\left(p_2'-p_2\right)x_2} \\
+\,\overline{u}_{s_1}(\vec{p_1'})\,\gamma^\mu v_{s_2}(\vec{p_2'})\,\overline{v}_{r_2}(\vec{p_2})\,\gamma^\nu u_{r_1}(\vec{p_1})\cdot
\end{aligned}$$

$$\cdot \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(q) \int \mathrm{d}^4 x_1 \mathrm{d}^4 x_2 \mathrm{e}^{-\mathrm{i}q(x_1-x_2)} \mathrm{e}^{\mathrm{i}\left(p_2'+p_1'\right)x_1} \mathrm{e}^{-\mathrm{i}(p_2+p_1)x_2} \Bigg\} . \tag{15}$$

The integrals over x_1 and x_2 are equal to two Dirac deltas, since

$$\int \frac{d^4 x}{(2\pi)^4} \mathrm{e}^{\mathrm{i}kx} = \delta\left(k\right) . \tag{16}$$

The two terms (without the spinors) become

$$\begin{aligned} &\int \mathrm{d}^4 q \, (2\pi)^4 \, \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(q) \delta\left(p_1' - p_1 - q\right) \delta\left(q - p_2' + p_2\right) \\ &= (2\pi)^4 \, \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}\left(p_2' - p_2\right) \delta\left[p_2' - p_2 - \left(p_1' - p_1\right)\right] , \end{aligned} \tag{17a}$$

$$\begin{aligned} &\int \mathrm{d}^4 q \, (2\pi)^4 \, \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(q) \delta\left(p_2' + p_1' - q\right) \delta\left(q - p_2 - p_1\right) \\ &= (2\pi)^4 \, \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}\left(p - 1 + p_2\right) \delta\left[p_2' + p_1' - \left(p_2 + p_1\right)\right] . \end{aligned} \tag{17b}$$

These two Dirac deltas guarantee that the processes respect the relativistic energy–momentum conservation. Finally, we have the result,

$$\begin{aligned} \langle f| \, S^{(2)}_{\text{Bhabha}} \, |i\rangle = \frac{(-\mathrm{i}e)^2}{(2\pi)^2} \frac{m^2}{\sqrt{E_{\vec{p_1}} E_{\vec{p_2}} E_{\vec{p_1'}} E_{\vec{p_2'}}}} \cdot \\ \cdot \Big\{ - \overline{u}_{s_1}\left(\vec{p_1'}\right) \gamma^\mu u_{r_1}\left(\vec{p_1}\right) \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(p_2' - p_2) \overline{v}_{s_2}\left(\vec{p_2'}\right) \gamma^\nu v_{r_2}\left(\vec{p_2}\right) \delta\left[p_2' - p_2 - \left(p_1' - p_1\right)\right] \\ + \overline{u}_{s_1}\left(\vec{p_1'}\right) \gamma^\mu v_{s_2}\left(\vec{p_2'}\right) \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(p_1 + p_2) \overline{v}_{r_2}\left(\vec{p_2}\right) \gamma^\nu u_{r_1}\left(\vec{p_1}\right) \delta\left[p_2' + p_1' - \left(p_2 + p_1\right)\right] \Big\} . \end{aligned} \tag{18}$$

We are finally in the position to define the two "Feynman amplitudes",

$$\mathcal{M}_a := -\left(-\mathrm{i}e\right)^2 \overline{u}_{s_1}\left(\vec{p_1'}\right) \gamma^\mu u_{r_1}\left(\vec{p_1}\right) \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(p_2' - p_2) \overline{v}_{s_2}\left(\vec{p_2'}\right) \gamma^\nu v_{r_2}\left(\vec{p_2}\right) , \tag{19a}$$

$$\mathcal{M}_b := \left(-\mathrm{i}e\right)^2 \overline{u}_{s_1}\left(\vec{p_1'}\right) \gamma^\mu v_{s_2}\left(\vec{p_2'}\right) \mathrm{i}\mathcal{D}^{\mathrm{F}}_{\mu\nu}(p_1 + p_2) \overline{v}_{r_2}\left(\vec{p_2}\right) \gamma^\nu u_{r_1}\left(\vec{p_1}\right) , \tag{19b}$$

which allow us to rewrite the S-matrix element as,

$$\begin{aligned} \langle f| \, S^{(2)}_{\text{Bhabha}} \, |i\rangle = \frac{1}{(2\pi)^2} \frac{m^2}{\sqrt{E_{\vec{p_1}} E_{\vec{p_2}} E_{\vec{p_1'}} E_{\vec{p_2'}}}} \cdot \\ \cdot \Big\{ \mathcal{M}_a \, \delta\left[p_2' - p_2 - \left(p_1' - p_1\right)\right] + \mathcal{M}_b \, \delta\left[p_2' + p_1' - \left(p_2 + p_1\right)\right] \Big\} . \end{aligned} \tag{20}$$

Now we substitute the expression for the photon propagator into the Feynman amplitudes and get,

$$\mathcal{M}_a := -\mathrm{i}e^2 \left[\overline{u}_{s_1}\left(\vec{p_1'}\right) \gamma^\mu u_{r_1}\left(\vec{p_1}\right)\right] \frac{\eta_{\mu\nu}}{\left(p_2' - p_2\right)^2 + \mathrm{i}\epsilon} \left[\overline{v}_{s_2}\left(\vec{p_2'}\right) \gamma^\nu v_{r_2}\left(\vec{p_2}\right)\right] \tag{21a}$$

$$= -\mathrm{i}e^2 \left[\overline{u}_{s_1}\left(\vec{p_1'}\right)\gamma^\mu u_{r_1}\left(\vec{p_1}\right)\right] \frac{1}{\left(p_2' - p_2\right)^2 + \mathrm{i}\epsilon} \left[\overline{v}_{s_2}\left(\vec{p_2'}\right)\gamma_\mu v_{r_2}\left(\vec{p_2}\right)\right], \quad (21b)$$

$$\mathcal{M}_b := \mathrm{i}e^2 \left[\overline{u}_{s_1}\left(\vec{p_1'}\right)\gamma^\mu v_{s_2}\left(\vec{p_2'}\right)\right] \frac{1}{\left(p_1 + p_2\right)^2 + \mathrm{i}\epsilon} \left[\overline{v}_{r_2}\left(\vec{p_2}\right)\gamma_\mu u_{r_1}\left(\vec{p_1}\right)\right]. \quad (21c)$$

Compare these formulas with (8.48) in M&S (we also have the spins and the names of the momenta are different). The Feynman amplitudes can be obtained much easier if one uses the "Feynman rules". Indeed, the calculation that we just completed is somewhat standard, and the steps that one has to perform for different scattering processes are all similar to what we have done. This allowed Feynman to define simple rules that can be used to avoid lengthy computations.

The two contributions $\mathcal{M}_a$ and $\mathcal{M}_b$ represent two distinct physical processes. The amplitude $\mathcal{M}_a$ describes the scattering between e^- and e^+ through the exchange of a virtual photon carrying momentum and energy (the so-called t-channel), whereas $\mathcal{M}_b$ describes the annihilation of the e^-e^+ pair into a photon, which then creates another e^-e^+ pair (s-channel). They correspond to the Feynman diagrams in Figure 1. Note that the Feynman diagrams are in bijective correspondence with the Feynman amplitudes via the Feynman rules [M&S, section 7.3, p. 118], therefore they are an handy way to write down integrals. *They are not drawings representing the actual physical process as it happens in spacetime.*

2 Scalar field self-interaction

7.3. A real scalar field $\phi(x)$, associated with a spin-zero boson B, is described by the Lagrangian density

$$\mathscr{L}(x) = \mathscr{L}_0(x) + \mathscr{L}_\mathrm{I}(x)$$

where $\mathscr{L}_0$ is the free-field density (3.4), and

$$\mathscr{L}_\mathrm{I}(x) = g[\phi(x)]^4/4!$$

describes an interaction of the field with itself, with g a real coupling constant. (Normal ordering of operators is assumed throughout.)

Write down the S-matrix expansion, and pick out the normal ordered term that gives rise to the BB scattering process

$$B(\mathbf{k}_1) + B(\mathbf{k}_2) \to B(\mathbf{k}_3) + B(\mathbf{k}_4)$$

in first-order perturbation theory. Draw the Feynman diagram representing this term, and show that the corresponding S-matrix element is given by

$$\langle k_3, k_4 | S^{(1)} | k_1, k_2 \rangle = (2\pi)^4 \delta^{(4)}(k_3 + k_4 - k_1 - k_2) \prod_i \left(\frac{1}{2V\omega_i} \right)^{1/2} \mathscr{M}$$

with the Feynman amplitude $\mathscr{M} = \mathrm{i}g$. [Note that $\mathscr{M}$ is independent of the boson four-momenta $k_i^\alpha \equiv (\omega_i, \mathbf{k}_i)$.]

Solution. We are considering a self-interaction of the type ϕ^4, which allows first-order processes. The first-order S-matrix is

$$\begin{aligned} S^{(1)} &= \frac{\mathrm{i}g}{4!} \int \mathrm{d}^4x \ \mathrm{N}\left[\phi^4\right] \\ &= \frac{\mathrm{i}g}{4!} \int \mathrm{d}^4x \ \mathrm{N}\left[\left(\phi^+(x) + \phi^-(x)\right)\left(\phi^+(x) + \phi^-(x)\right)\left(\phi^+(x) + \phi^-(x)\right)\left(\phi^+(x) + \phi^-(x)\right)\right]. \end{aligned} \tag{22}$$

In the previous exercise, we saw that the general computation is quite lengthy. Therefore, now, we simplify our analysis by doing some considerations.

First, we want terms involving two annihilation and two creation operators. This can be achieved in six ways, namely,

$$\begin{gathered} \phi^+\phi^+\phi^-\phi^-, \quad \phi^+\phi^-\phi^+\phi^-, \quad \phi^+\phi^-\phi^-\phi^+, \\ \phi^-\phi^-\phi^+\phi^+, \quad \phi^-\phi^+\phi^-\phi^+, \quad \phi^-\phi^+\phi^+\phi^-. \end{gathered} \tag{23}$$

Note that these are scalar fields, so they all commute when we perform the normal ordering. All these terms become $\phi^-\phi^-\phi^+\phi^+$ after normal ordering. For each term, we have 2 ways to assign momenta for the initial state and 2 ways for the final state, which means 4 ways in total. So we have a factor of $6 \times 4 = 24 = 4!$ in front of the S-matrix element, which exactly cancel the prefactor $\frac{1}{4!}$. Then,

$$\tilde{S}^{(1)} = \mathrm{i}g \int \mathrm{d}^4x\, \phi^-(x)\phi^-(x)\phi^+(x)\phi^+(x). \tag{24}$$

The quantity $\tilde{S}^{(1)}$ is the part of $S^{(1)}$ describing the process we care about.

Now we insert the explicit expression for ϕ [eq.(3.7) in M&S],

$$\phi^+(x) = \int \frac{d^4q}{2\pi^{3/2}} \sqrt{\frac{1}{2E_{\vec{q}}}} a\left(\vec{k}\right) \mathrm{e}^{-\mathrm{i}qx}, \quad \phi^-(x) = \int \frac{d^4q}{2\pi^{3/2}} \sqrt{\frac{1}{2E_{\vec{q}}}} a^\dagger\left(\vec{k}\right) \mathrm{e}^{\mathrm{i}qx}. \tag{25}$$

The S-matrix element becomes

$$\begin{aligned}\langle f|\, \tilde{S}^{(1)} \,|i\rangle = \mathrm{i}g \int \mathrm{d}^4x\, \Pi_j \left(\frac{1}{\sqrt{2E_{\vec{q_j}}}}\right) \mathrm{e}^{-\mathrm{i}q_1x}\mathrm{e}^{-\mathrm{i}q_2x}\mathrm{e}^{\mathrm{i}q_1'x}\mathrm{e}^{\mathrm{i}q_2'x} \langle f|\, a^\dagger\left(\vec{q_2'}\right) a^\dagger\left(\vec{q_1'}\right) a\left(\vec{q_2}\right) a\left(\vec{q_1}\right) |i\rangle \\ \mathrm{i}g\Pi_j \left(\frac{1}{\sqrt{2E_{\vec{q_j}}}}\right) \delta\left(q_1' + q_2' - q_1 - q_2\right) (2\pi)^4 \langle f|\, a^\dagger\left(\vec{q_2'}\right) a^\dagger\left(\vec{q_1'}\right) a\left(\vec{q_2}\right) a\left(\vec{q_1}\right) |i\rangle,\end{aligned} \tag{26}$$

with $|f\rangle = a^\dagger\left(\vec{q_3}\right) a^\dagger\left(\vec{q_4}\right)|0\rangle$ and $|i\rangle = a^\dagger\left(\vec{q_1}\right) a^\dagger\left(\vec{q_2}\right)|0\rangle$.[1] This implies that

$$\langle f|\, a^\dagger\left(\vec{q_2'}\right) a^\dagger\left(\vec{q_1'}\right) a\left(\vec{q_2}\right) a\left(\vec{q_1}\right) |i\rangle = \langle 0|0\rangle = 1. \tag{27}$$

Hence,

$$\langle f|\, \tilde{S}^{(1)} \,|i\rangle = \mathrm{i}g\, \Pi_j \left(\frac{1}{\sqrt{2E_{\vec{q_j}}}}\right) \delta\left(q_1' + q_2' - q_1 - q_2\right) (2\pi)^4, \tag{28}$$

from which we can extract the Feynman amplitude

$$\mathcal{M} = \mathrm{i}g. \tag{29}$$

[1] Note that we skipped many steps here. Why are the initial and final momenta equal to the momenta q_j coming from the fields?

3 Interaction with an external static potential

7.5. A real scalar field $\phi(x)$ is described by the Lagrangian density

$$\mathscr{L}(x) = \mathscr{L}_0(x) + \mu U(\mathbf{x})\phi^2(x),$$

where $\mathscr{L}_0$ is the free-field Lagrangian density (3.4), and $U(\mathbf{x})$ *is* a static external potential.

Derive the equation of motion

$$(\Box + \mu^2)\phi(x) = 2\mu U(\mathbf{x})\phi(x).$$

Show that, in lowest order, the S-matrix element for an incoming boson, with momentum $k_i = (\omega_i, \mathbf{k}_i)$, to be scattered to a state with momentum $k_f = (\omega_f, \mathbf{k}_f)$, is given by

$$\left\langle \mathbf{k}_f \middle| S^{(1)} \middle| \mathbf{k}_i \right\rangle = \frac{\mathrm{i}2\pi\delta(\omega_f - \omega_i)}{(2V\omega_i)^{1/2}(2V\omega_f)^{1/2}} 2\mu\, \widetilde{U}(\mathbf{k}_f - \mathbf{k}_i)$$

where

$$\widetilde{U}(\mathbf{q}) = \int \mathrm{d}^3\mathbf{x}\, U(\mathbf{x}) \mathrm{e}^{-\mathrm{i}\mathbf{q}\cdot\mathbf{x}}.$$

This type of problem, with a static external potential, will be considered further in Chapter 8.

Solution. To derive the equations of motion, we know that $\mathscr{L}_0$ gives the Klein–Gordon equation. The variation of $\mathscr{L}_I$ gives,

$$\begin{aligned}\delta\left[\int \mathrm{d}^4x \mu U(\vec{x})\phi(x)^2\right] &= \mu \int \mathrm{d}^4x U(\vec{x}) \left(\phi(x)\delta\phi(x) + \delta\phi(x)\cdot\phi(x)\right) \\ &= 2\mu \int \mathrm{d}^4x U(\vec{x})\phi(x)\delta\phi(x).\end{aligned} \tag{30}$$

The Euler–Lagrange equations are,

$$\left(\Box + \mu^2\right)\phi(x) = 2\mu U(\vec{x})\phi(x). \tag{31}$$

We consider a first-order process, hence the S-matrix is

$$S^{(1)} = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{N}\left[\mu U(\vec{x})\phi(x)^2\right]. \tag{32}$$

In the initial and final states we have one particle, so we need the terms having only one creation and one annihilation operator.

$$S^{(1)} = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{N}\left[\mu U(\vec{x})\left(\phi^+(x) + \phi^-(x)\right)\left(\phi^+(x) + \phi^-(x)\right)\right]. \tag{33}$$

We can have $\phi^-\phi^+$ and $\phi^+\phi^-$, both normal ordered to $\phi^-\phi^+$. The other terms would lead to scalar products between orthogonal states. The computation reads,

$$
\begin{aligned}
\langle f| S^{(1)} |i\rangle &= 2\mathrm{i}\int \mathrm{d}^4x\mu U(\vec{x})\,\langle f|\,\phi^-(x)\phi^+(x)\,|i\rangle \\
&= 2\mathrm{i}\mu\int \mathrm{d}^4xU(\vec{x})\Pi_j\left(\frac{1}{\sqrt{2E_{\vec{k_j}}}}\right)\mathrm{e}^{\mathrm{i}k_f x}\mathrm{e}^{-\mathrm{i}k_i x} \\
&= 2\mathrm{i}\mu\Pi_j\left(\frac{1}{\sqrt{2E_{\vec{k_j}}}}\right)\int \mathrm{d}^4xU(\vec{x})\mathrm{e}^{\mathrm{i}k_f x}\mathrm{e}^{-\mathrm{i}k_i x} \\
&= 2\mathrm{i}\mu\Pi_j\left(\frac{1}{\sqrt{2E_{\vec{k_j}}}}\right)2\pi\delta\left(\omega_f-\omega_i\right)\int \mathrm{d}^3xU(\vec{x})\mathrm{e}^{\mathrm{i}(\vec{k_f}-\vec{k_i})\vec{x}} \\
&= 2\mathrm{i}\mu\Pi_j\left(\frac{1}{\sqrt{2E_{\vec{k_j}}}}\right)2\pi\delta\left(\omega_f-\omega_i\right)\tilde{U}(\vec{k_f}-\vec{k_i}) \\
&= \frac{2\pi\mathrm{i}\,\delta\left(\omega_f-\omega_i\right)}{(2\omega_f)^{1/2}(2\omega_i)^{1/2}}2\mu\tilde{U}\left(\vec{k_f}-\vec{k_i}\right). \qquad (34)
\end{aligned}
$$

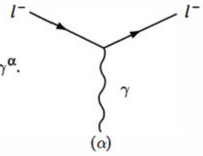
4 Feynman rules for QED

The Feynman rules are taken from M&S, Section 7.3 and Appendix B.

(iv) *The Feynman rules for QED.* The Feynman amplitude for a given graph in QED is obtained from the Feynman rules of Sections 7.3 and 8.7. Some of the Feynman rules for charged leptons $l^{\pm}$ apply with trivial changes to neutral leptons ν_l and $\bar{\nu}_l$. To avoid lengthy repetition later, we write these rules at once for both charged and neutral leptons.

1. For each QED vertex, write a factor

$ie\gamma^{\alpha}$.

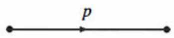


2. For each internal photon line, labelled by the momentum k, write a factor

$$iD_{F\alpha\beta}(k) = i\frac{-g_{\alpha\beta}}{k^2 + i\varepsilon}. \tag{7.47}$$

3. For each internal lepton line, labelled by the momentum p, write a factor

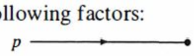
$$iS_F(p) = i\frac{1}{\not{p} - m + i\varepsilon}. \tag{7.48}$$



Here m stands for the mass m_l, or m_{ν_l} of the particular lepton considered.

4. For each external line, write one of the following factors:

(a) for each initial lepton l^- or ν_l : $u_r(\mathbf{p})$ (7.49a)



(b) for each final lepton l^- or ν_l : $\bar{u}_r(\mathbf{p})$ (7.49b)

(c) for each initial lepton l^+ or $\bar{\nu}_l$: $\bar{v}_r(\mathbf{p})$ (7.49c)

(d) for each final lepton l^+ or $\bar{\nu}_l$: $v_r(\mathbf{p})$ (7.49d)

(e) for each initial photon: $\varepsilon_{r\alpha}(\mathbf{k})$ (7.49e)

(f) for each final photon:[2] $\varepsilon_{r\alpha}(\mathbf{k})$ (7.49f)

In Eqs. (7.49) $\mathbf{p}$ and $\mathbf{k}$ denote the three-momenta of the external particles, and $r (= 1, 2)$ their spin and polarization states.

5. The spinor factors (γ-matrices, S_F-functions, four-spinors) for each fermion line are ordered so that, reading from right to left, they occur in the same sequence as following the fermion line in the direction of its arrows.
6. For each closed fermion loop, take the trace and multiply by a factor (-1).
7. The four-momenta associated with the lines meeting at each vertex satisfy energy–momentum conservation. For each four-momentum q which is not fixed by energy–momentum conservation carry out the integration $(2\pi)^{-4}\int d^4q$. One such integration with respect to an internal momentum variable q occurs for each closed loop.
8. Multiply the expression by a phase factor δ_p which is equal to $+1$ (-1) if an even (odd) number of interchanges of neighbouring fermion operators is required to write the fermion operators in the correct normal order.

To allow for the interaction with an *external static electromagnetic field* $A_{e\alpha}(x)$:

(a) In Eq. (8.1), relating $\mathscr{M}$ to S_{fi} make the replacement

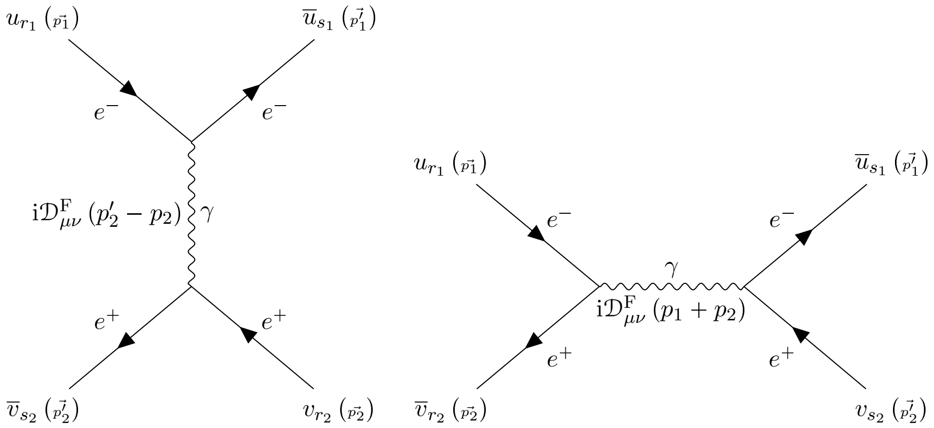
$$(2\pi)^4\delta^{(4)}\left(\sum p_f' - \sum p_i\right) \to (2\pi)\delta\left(\sum E_f' - \sum E_i\right) \tag{8.89}$$

(b) Add the following Feynman rule:

9. For each interaction of a charged particle with an external static field $A_e(\mathbf{x})$, write a factor

$$A_{e\alpha}(\mathbf{q}) = \int d^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} A_{e\alpha}(\mathbf{x}) \tag{8.90}$$

Here $\mathbf{q}$ is the momentum transferred from the field source (x) to the particle.



(e) Elastic scattering, t-channel, $\mathcal{M}_a$. (f) Pair annihilation and creation, s-channel, $\mathcal{M}_b$.

Figure 1: Feynman diagrams corresponding to the two Feynman amplitudes $\mathcal{M}_a$ and $\mathcal{M}_b$ of the Bhabha scattering $e^- e^+ \to e^- e^+$. Time goes from left to right.

According to these rules, the two Feynman amplitudes for Bhabha scattering correspond to the Feynman diagrams in Figure 1.

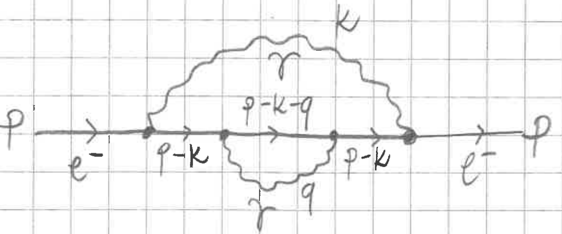
$$\mathcal{M}_a = -\mathrm{i}e^2 \left[\overline{u}_{s_1}(\vec{p_1'})\, \gamma^\mu u_{r_1}(\vec{p_1})\right] \frac{1}{(p_2' - p_2)^2 + \mathrm{i}\epsilon} \left[\overline{v}_{s_2}(\vec{p_2'})\, \gamma_\mu v_{r_2}(\vec{p_2})\right], \tag{35a}$$

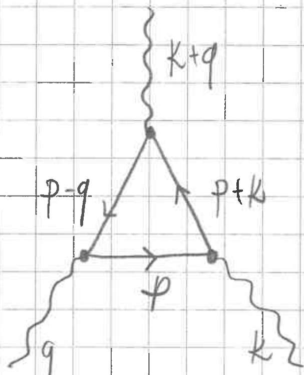
$$\mathcal{M}_b = \mathrm{i}e^2 \left[\overline{u}_{s_1}(\vec{p_1'})\, \gamma^\mu v_{s_2}(\vec{p_2'})\right] \frac{1}{(p_1 + p_2)^2 + \mathrm{i}\epsilon} \left[\overline{v}_{r_2}(\vec{p_2})\, \gamma_\mu u_{r_1}(\vec{p_1})\right]. \tag{35b}$$

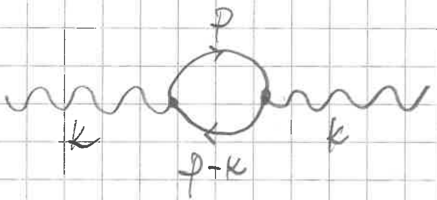
Write the Feynman amplitudes corresponding to the following Feynman diagrams, using the Feynman rules.

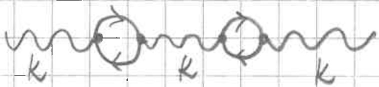
(a) 

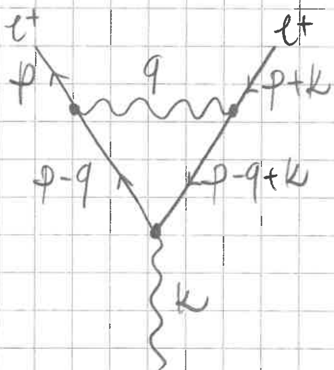
iM = ?

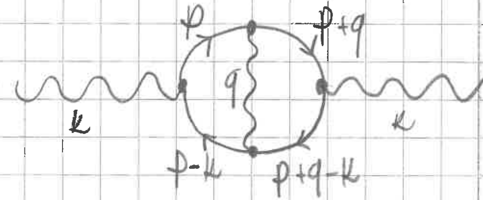
(b) 

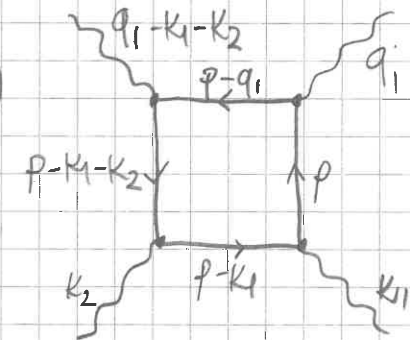
(c) 

(f) 

(g) 

(d) 

(h) 

(i) 

(e)