Tutorial on cross sections, Problems

2014-11-18 15:15-17:00 FB41, v1 [printed: 2014-11-12]

1 Problem: Cross-Section in Lab Frame

Let $p_i = (E_i, \mathbf{p}_i)$, $i = 1, 2$ be the four-momenta of two colliding particles with masses m_i, and $p_i' = (E_i', \mathbf{p}_i')$, $f = 1, 2$ the their outgoing four-momenta, as on Fig. 1. Compute the cross section for this process in the lab frame. The second particle is nearly stationary (a target).

2 Problem: e^-e^+ scattering in EM field

Calculate the cross section per unit volume for the creation of electron–positron pairs by the electromagnetic potential $A_\mu = (0, 0, a\mathrm{e}^{-\mathrm{i}\omega t}, 0)$, where ω and a are constants.

3 Spin-sums Lemma *(Casimir's trick for spin-sums)*

Lemma. Let $u_r(\mathbf{p})$ and $\bar{u}_{r'}(\mathbf{p}')$ be two spinors which completely specify the momenta and spins of an electron with the mass m in the initial and final states. Let further A and B be two 4×4 matrices built up out of γ-matrices. Consider a sum

$$X = \sum_{r,r'} \left(\bar{u}_{r'}(\mathbf{p}')\, A\, u_r(\mathbf{p})\right) \left(\bar{u}_{r'}(\mathbf{p}')\, B\, u_r(\mathbf{p})\right)^\dagger . \tag{3.1}$$

Then it follows

$$X = \frac{1}{4m^2} \mathrm{Tr}\left[(\not{p}' + m)\, A\, (\not{p} + m)\, \tilde{B} \right] \tag{3.2}$$

where $\tilde{B} := \gamma^0 B^\dagger \gamma^0$.

Similarly for the other different combinations of spinors, it holds

$$\sum_{r,r'} \left(\bar{v}_{r'}(\mathbf{p}')\, A\, v_r(\mathbf{p})\right) \left(\bar{v}_{r'}(\mathbf{p}')\, B\, v_r(\mathbf{p})\right)^\dagger = \frac{1}{4m^2} \mathrm{Tr}\left[(\not{p}' - m)\, A\, (\not{p} - m)\, \tilde{B} \right] , \tag{3.3}$$

$$\sum_{r,r'} \left(\bar{u}_{r'}(\mathbf{p}')\, A\, v_r(\mathbf{p})\right) \left(\bar{u}_{r'}(\mathbf{p}')\, B\, v_r(\mathbf{p})\right)^\dagger = \frac{1}{4m^2} \mathrm{Tr}\left[(\not{p}' + m)\, A\, (\not{p} - m)\, \tilde{B} \right] , \tag{3.4}$$

$$\sum_{r,r'} \left(\bar{v}_{r'}(\mathbf{p}')\, A\, u_r(\mathbf{p})\right) \left(\bar{v}_{r'}(\mathbf{p}')\, B\, u_r(\mathbf{p})\right)^\dagger = \frac{1}{4m^2} \mathrm{Tr}\left[(\not{p}' - m)\, A\, (\not{p} + m)\, \tilde{B} \right] . \tag{3.5}$$

Prove these identities.