

4 Feynman diagrams

Using the Feynman rules (see Appendix) we obtain the expression for the corresponding amplitudes. (In the following expressions we drop external lines.)

a)

$$\begin{aligned}
 i\mathcal{M} &= \text{Diagram: a horizontal line with momentum } p \text{ entering from the left and } p-k \text{ exiting to the right. A wavy loop with momentum } k \text{ is attached to the line.} \\
 &= (ie)^2 \int \frac{d^4k}{(2\pi)^4} \left(\gamma_\nu \frac{1}{\not{p} - \not{k} - m + i\epsilon} \gamma_\mu \frac{g^{\mu\nu}}{k^2 + i\epsilon} \right)
 \end{aligned}$$

b)

$$\begin{aligned}
 i\mathcal{M} &= \text{Diagram: a horizontal line with momentum } p \text{ entering from the left. A loop with momentum } k \text{ is attached to the line, and another loop with momentum } q \text{ is attached to the bottom of the } k \text{ loop.} \\
 &= i(ie)^4 \int \int \frac{d^4k}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \left(\gamma^\mu \frac{1}{\not{p} - \not{k} - m + i\epsilon} \gamma^\sigma \right. \\
 &\quad \times \frac{1}{\not{p} - \not{k} - \not{q} - m + i\epsilon} \gamma^\sigma \frac{1}{\not{p} - \not{k} - m + i\epsilon} \\
 &\quad \left. \times \gamma_\mu \frac{1}{k^2 + i\epsilon} \frac{1}{q^2 + i\epsilon} \right)
 \end{aligned}$$

c)

$$\begin{aligned}
 i\mathcal{M} &= \text{Diagram: a triangle loop with vertices labeled } p-q, p+k, \text{ and } p. \text{ External wavy lines are attached to each vertex, labeled } q, k, \text{ and } p \text{ respectively.} \\
 &= -(ie)^3 i^3 \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[\gamma^\nu \frac{1}{\not{p} - \not{q} - m + i\epsilon} \gamma^\rho \right. \\
 &\quad \left. \times \frac{1}{\not{p} + \not{k} - m + i\epsilon} \gamma^\mu \frac{1}{\not{p} - m + i\epsilon} \right]
 \end{aligned}$$

d)

$$\begin{aligned}
& \text{Diagram: A vertex with two incoming lines (left and right) and one outgoing line (bottom). The left line has momentum p, the right line has momentum $p-q+k$, and the bottom line has momentum k. A wavy line with momentum q connects the two incoming lines.} \\
i\mathcal{M} = & \\
& = i(\text{ie})^3 \int \frac{d^4 p}{(2\pi)^4} \left(\gamma^\nu \frac{1}{\not{p} + \not{k} - \not{q} - m + i\epsilon} \right. \\
& \qquad \qquad \qquad \left. \times \gamma^\rho \frac{1}{\not{p} - \not{q} - m + i\epsilon} \gamma_\nu \frac{1}{\not{q}^2 + i\epsilon} \right)
\end{aligned}$$

e)

$$\begin{aligned}
& \text{Diagram: A vertex with two incoming lines (left and right) and one outgoing line (bottom). The left line has momentum p, the right line has momentum p_1+q, and the bottom line has momentum p_1. A wavy line with momentum q connects the two incoming lines. A loop with momentum k is attached to the left line, and another loop with momentum k_1 is attached to the bottom line.} \\
i\mathcal{M} = & \\
& = (\text{ie})^7 i^6 (-i)^3 \int \int \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \\
& \times \left[\gamma^\nu \frac{1}{\not{p}_1 + \not{q} - m + i\epsilon} \gamma^\alpha \frac{1}{\not{q} - m + i\epsilon} \gamma^\mu \right. \\
& \times \frac{g^{\mu\rho}}{(p-q)^2 + i\epsilon} \frac{g^{\sigma\nu}}{(p-q)^2 + i\epsilon} \\
& \times \text{tr} \left(\frac{1}{\not{k} - m + i\epsilon} \gamma^\sigma \frac{1}{\not{p} - \not{q} + \not{k} - m + i\epsilon} \gamma^\rho \right) \\
& \left. \times \frac{g^{\alpha\beta}}{p_1^2 + i\epsilon} \text{tr} \left(\frac{1}{\not{p}_1 + \not{k}_1 - m + i\epsilon} \gamma^\delta \frac{1}{\not{k}_1 - m + i\epsilon} \gamma^\beta \right) \right]
\end{aligned}$$

f)

$$\begin{aligned}
-i\Pi^{\mu\nu}(k) &= \text{Diagram: a circle with two wavy external lines. The left line has momentum k entering, and the right line has momentum k exiting. The top arc has momentum p and the bottom arc has momentum $p-k$} \\
&= (ie)^2 \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[\frac{1}{\not{p} - \not{k} - m + i\epsilon} \gamma^\nu \frac{1}{\not{p} - m + i\epsilon} \gamma^\mu \right]
\end{aligned}$$

g)

$$-i\mathcal{M} = \text{Diagram: two circles connected by a wavy line. The left circle has momentum k entering, and the right circle has momentum k exiting. The wavy line connects the two circles.} = (-i)\Pi^{\mu\nu}(k) \frac{-ig_{\nu\rho}}{k^2 + i\epsilon} (-i)\Pi^{\rho\sigma}(k)$$

h)

$$\begin{aligned}
-i\mathcal{M} &= \text{Diagram: a circle with two wavy external lines. The left line has momentum k entering, and the right line has momentum k exiting. The top arc has momentum p and the bottom arc has momentum $p+q-k$. The circle is split by a wavy line with momentum q} \\
&= -i^4(-i)(ie)^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \text{tr} \left[\frac{1}{\not{p} - \not{k} - m + i\epsilon} \gamma^\sigma \right. \\
&\quad \times \frac{1}{\not{p} + \not{q} - \not{k} - m + i\epsilon} \gamma^\nu \frac{1}{\not{p} + \not{q} - m + i\epsilon} \gamma^\rho \\
&\quad \left. \times \frac{1}{\not{p} - m + i\epsilon} \gamma^\mu \right] \frac{g_{\rho\sigma}}{q^2 + i\epsilon}
\end{aligned}$$

i)

$$\begin{aligned}
i\mathcal{M} &= \text{Diagram: a square loop with four wavy external lines. The top line has momentum $p-q_1$ and momentum q_1 entering. The bottom line has momentum $p-k_1$ and momentum k_1 entering. The left line has momentum $p-k_1-k_2$ and momentum k_2 entering. The right line has momentum p and momentum p entering. The loop is oriented clockwise.} \\
&= -(ie)^4 \int \frac{d^4p}{(2\pi)^4} \text{tr} \left[\frac{1}{\not{p} - \not{k}_1 - m + i\epsilon} \gamma^\mu \frac{1}{\not{p} - \not{k}_1 - \not{k}_2 - m + i\epsilon} \right. \\
&\quad \left. \times \gamma^\sigma \frac{1}{\not{p} - \not{q}_1 - m + i\epsilon} \gamma^\rho \frac{1}{\not{p} - m + i\epsilon} \gamma^\nu \right]
\end{aligned}$$