Tutorial 14

FK8027 - Quantum Field Theory

Monday 18$^{\text{th}}$ February, 2019

Topics for today

- Brout–Englert–Higgs (BEH) mechanism for the massive real vector bosons W_1^μ, W_2^μ and the real vector bosons W_3^μ, B^μ; diagonalization to the mass eigenstates: the massive real vector boson Z^μ and the massless real vector boson A^μ (photon)
- Feynman rules for two first-order electroweak processes
- The discovery of the Higgs boson
- Mention of the hierarchy problem

Notation. All the fields depend on the spacetime coordinates, but we shall omit them for readability.

1 BEH mechanism

The Lagrangian density for the Higgs isospinor doublet is[1]

$$\mathscr{L}^{\mathrm{H}} = \left[D^\mu \Phi\right]^\dagger D_\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda \left[\Phi^\dagger \Phi\right]^2 , \tag{1}$$

with $\mu^2 < 0$ and $\lambda > 0$. The Higgs isospinor doublet can be written

$$\Phi = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1 + \mathrm{i}\,\eta_2 \\ v + \sigma + \mathrm{i}\eta_3 \end{pmatrix} \tag{2}$$

where $v = \sqrt{2}\,\langle 0|\,\phi_b\,|0\rangle = \sqrt{-\mu^2/\lambda} > 0$, σ is the real scalar Higgs field, and η_i are the gauge-dependent (hence nonphysical) real scalar fields.[2] We can always find a gauge transformation setting the η_i to zero, called the "unitary gauge" (Sec. 19.1 in M&S), i.e.,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \sigma \end{pmatrix} \tag{3}$$

[1] Note that the fact that the Higgs isospinor is a doublet implies that it has isospin 1/2 (why?).

[2] They are three because the standard electroweak theory breaks SU(2), which has three generators. The freedom to fix the η_i is exactly the freedom to choose a specific gauge transformation in SU(2), which requires three parameters to be specified.

In the standard electroweak theory, the mass terms for the gauge vector bosons arise from the interaction with the Higgs real scalar field. This appears in the kinetic term for the Higgs isospinor doublet Φ,

$$[D^\mu \Phi]^\dagger D_\mu \Phi = \left[\left(\partial^\mu + \mathrm{i}\frac{g}{2}\tau^j W_j^\mu + \mathrm{i}g' Y B^\mu\right)\Phi\right]^\dagger \left(\partial_\mu + \mathrm{i}\frac{g}{2}\tau_j W_\mu^j + \mathrm{i}g' Y B_\mu\right)\Phi, \tag{4}$$

where τ^j are the Pauli matrices, Y is the hypercharge operator, W_j^μ are the components of the SU(2) gauge real vector bosons in the Pauli matrices' basis, B^μ is the $\mathrm{U(1)_Y}$ gauge real vector boson, g and g' are SU(2) and $\mathrm{U(1)_Y}$ couplings.

Expanding (4) we get

$$\begin{aligned}
[D^\mu \Phi]^\dagger D_\mu \Phi &= \left[\left(\partial^\mu + \mathrm{i}\frac{g}{2}\tau^j W_j^\mu + \mathrm{i}g' Y B^\mu\right)\Phi\right]^\dagger \left(\partial_\mu + \mathrm{i}\frac{g}{2}\tau_j W_\mu^j + \mathrm{i}g' Y B_\mu\right)\Phi \\
&= \partial^\mu \Phi^\intercal \left(\partial_\mu + \mathrm{i}\frac{g}{2}\tau_j W_\mu^j + \mathrm{i}g' Y^\dagger B_\mu\right)\Phi \; + \; \left[\left(\partial^\mu + \mathrm{i}\frac{g}{2}\tau^j W_j^\mu + \mathrm{i}g' Y B^\mu\right)\Phi\right]^\dagger \partial_\mu \Phi \\
&+ \left(-\mathrm{i}\frac{g}{2}\Phi^\intercal \tau^j W_j^\mu\right)\left(\mathrm{i}\frac{g}{2}\tau_j W_\mu^j \Phi\right) \; + \; \left(-\mathrm{i}\frac{g}{2}\Phi^\intercal \tau^j W_j^\mu\right)\left(\mathrm{i}g' Y B_\mu \Phi\right) \\
&+ \left(-\mathrm{i}g' \Phi^\intercal Y^\dagger B^\mu\right)\left(\mathrm{i}\frac{g}{2}\tau_j W_\mu^j \Phi\right) \; + \; \left(-\mathrm{i}g' \Phi^\intercal Y^\dagger B^\mu\right)\left(\mathrm{i}g' Y B_\mu \Phi\right).
\end{aligned} \tag{5}$$

We remind that the Pauli matrices are self-adjoint. We do not need to consider all these terms. The first line is not quadratic in the vector bosons, hence it does not give rise to their mass terms. Rather, it describes interactions between the Higgs field and the bosons. So we consider only the last two lines, which we denote D. We can act with the hypercharge operator on the Higgs field,

$$Y\,\Phi = \frac{1}{2}\,\Phi \quad \text{[M\&S, between (18.39) and (18.40)]}. \tag{6}$$

In addition, the mass terms only involve the vacuum expectation value v, and not the Higgs field σ (the terms involving σ are other interactions). Therefore we can simplify D as follows,

$$\begin{aligned}
D = \frac{g^2 v^2}{8}\begin{pmatrix} 0 & 1 \end{pmatrix} \Bigg\{ & \begin{pmatrix} W_3^\mu & W_1^\mu - \mathrm{i}W_2^\mu \\ W_1^\mu + \mathrm{i}W_2^\mu & -W_3^\mu \end{pmatrix}\begin{pmatrix} W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2 \\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3 \end{pmatrix} \\
& + \frac{g'}{g}\left[\begin{pmatrix} W_3^\mu & W_1^\mu - \mathrm{i}W_2^\mu \\ W_1^\mu + \mathrm{i}W_2^\mu & -W_3^\mu \end{pmatrix}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}\right. \\
& + \left.\begin{pmatrix} B^\mu & 0 \\ 0 & B^\mu \end{pmatrix}\begin{pmatrix} W_\mu^3 & W_\mu^1 - \mathrm{i}W_\mu^2 \\ W_\mu^1 + \mathrm{i}W_\mu^2 & -W_\mu^3 \end{pmatrix}\right] \\
& + \left(\frac{g'}{g}\right)^2 \begin{pmatrix} B^\mu & 0 \\ 0 & B^\mu \end{pmatrix}\begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}\Bigg\}\begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\end{aligned} \tag{7}$$

Performing the matrix products leads to

$$
D = \frac{g^2 v^2}{8} \begin{pmatrix} 0 & 1 \end{pmatrix} \left\{ \begin{pmatrix} W_i^\mu W_\mu^i & 0 \\ 0 & W_i^\mu W_\mu^i \end{pmatrix} + \left(\frac{g'}{g}\right)^2 \begin{pmatrix} B^\mu B_\mu & 0 \\ 0 & B^\mu B_\mu \end{pmatrix} \right.
$$

$$
\left. +2 \frac{g'}{g} \begin{pmatrix} W_3^\mu B_\mu & W_1^\mu B_\mu - \mathrm{i} W_2^\mu B_\mu \\ W_1^\mu B_\mu + \mathrm{i} W_2^\mu B_\mu & -W_3^\mu B_\mu \end{pmatrix} \right\} \begin{pmatrix} 0 \\ 1 \end{pmatrix} . \tag{8}
$$

Lastly, we do the matrix product with the Higgs doublet

$$
D = \frac{g^2 v^2}{8} \left[W_i^\mu W_\mu^i - 2 \frac{g'}{g} \boxed{W_3^\mu B_\mu} + \left(\frac{g'}{g}\right)^2 B^\mu B_\mu \right] , \tag{9}
$$

and we get the mass terms for the vector bosons W_1^μ and W_2^μ.

We note that W_3^μ and B^μ are not mass eigenstates of the Hamiltonian, because there is a cross term coupling them (the one in the box). As explained in M&S [see eq. (17.45)], we can rotate the states by the weak mixing angle (usually called the Weinberg angle) and get the two mass eigenstates Z^μ (electrically neutral massive vector boson) and A^μ (photon).

$$
W_3^\mu = \cos(\theta_\mathrm{W})\, Z^\mu + \sin(\theta_\mathrm{W})\, A_\mu, \tag{10a}
$$

$$
B^\mu = -\sin(\theta_\mathrm{W})\, Z^\mu + \cos(\theta_\mathrm{W})\, A_\mu. \tag{10b}
$$

θ_W can be determined experimentally from electron–neutrino scattering or Möller scattering.

We now plug (10) in (9), and get

$$
\begin{aligned}
D = \frac{g^2 v^2}{8} \Bigg\{ & Z^\mu Z_\mu \left[\cos(\theta_\mathrm{W})^2 + 2 \frac{g'}{g} \cos(\theta_\mathrm{W}) \sin(\theta_\mathrm{W}) + \left(\frac{g'}{g}\right)^2 \sin(\theta_\mathrm{W})^2 \right] \\
& + Z^\mu A_\mu \Bigg[2 \cos(\theta_\mathrm{W}) \sin(\theta_\mathrm{W}) - 2 \frac{g'}{g} \left(\cos(\theta_\mathrm{W})^2 - \sin(\theta_\mathrm{W})^2 \right) \\
& \quad -2 \left(\frac{g'}{g}\right)^2 \sin(\theta_\mathrm{W}) \cos(\theta_\mathrm{W}) \Bigg] \\
& + A^\mu A_\mu \left[\sin(\theta_\mathrm{W})^2 - 2 \frac{g'}{g} \sin(\theta_\mathrm{W}) \cos(\theta_\mathrm{W}) + \left(\frac{g'}{g}\right)^2 \cos(\theta_\mathrm{W})^2 \right] \Bigg\} .
\end{aligned} \tag{11}
$$

The first term is a perfect square

$$
\cos(\theta_\mathrm{W})^2 + 2 \frac{g'}{g} \cos(\theta_\mathrm{W}) \sin(\theta_\mathrm{W}) + \left(\frac{g'}{g}\right)^2 \sin(\theta_\mathrm{W})^2 = \left[\cos(\theta_\mathrm{W}) + \frac{g'}{g} \sin(\theta_\mathrm{W}) \right]^2 . \tag{12}
$$

Now we use eq (17.47) in M&S,

$$\frac{g'}{g} = \tan(\theta_{\mathrm{W}}), \tag{13}$$

which is justified in M&S by the requirement that the field A_μ is the photon and couples to matter as in QED. We get

$$\begin{aligned} D = \frac{g^2 v^2}{8} \Bigg\{ & Z^\mu Z_\mu \left[\frac{\left(\cos(\theta_{\mathrm{W}})^2 + \sin(\theta_{\mathrm{W}})^2\right)^2}{\cos(\theta_{\mathrm{W}})^2} \right] \\ & + 2 Z^\mu A_\mu \left[\cos(\theta_{\mathrm{W}}) \sin(\theta_{\mathrm{W}}) - \cos(\theta_{\mathrm{W}}) \sin(\theta_{\mathrm{W}}) + \frac{\sin(\theta_{\mathrm{W}})^3}{\cos(\theta_{\mathrm{W}})} - \frac{\sin(\theta_{\mathrm{W}})^3}{\cos(\theta_{\mathrm{W}})} \right] \\ & + A^\mu A_\mu \left[\sin(\theta_{\mathrm{W}})^2 - 2\sin(\theta_{\mathrm{W}})^2 + \sin(\theta_{\mathrm{W}})^2 \right] \Bigg\} \\ = {} & \frac{g^2 v^2}{8\cos(\theta_{\mathrm{W}})^2} Z^\mu Z_\mu . \end{aligned} \tag{14}$$

Finally, we get a mass term for Z^μ only, which means that the photon A^μ is massless. Indeed, setting the Higgs hypercharge to 1/2 makes its electric charge to vanish. In this way, $U(1)_{\mathrm{EM}}$ is not broken and the photon stays massless.

The mass of the bosons are

$$m_W = \frac{gv}{2}, \quad m_Z = \frac{m_W}{\cos(\theta_{\mathrm{W}})},$$
$$\sin(\theta_{\mathrm{W}})^2 = 0.23122 \pm 0.00015 \text{ [M\&S, eq. (19.13c)]}.$$

2 Feynman rules for two first-order electroweak processes

Contrary to QED, where *every* vertex gives a factor $\mathrm{i}q\gamma^\mu$ (with q the appropriate charge), in electroweak theory we can have 18 basic vertices, listed in Appendix B in M&S (see also the standard electroweak Lagrangian in Sec. 19.1). We are going to compute explicitly the Feynman rules for two vertices, namely (B.11) and (B.3).

(B.11) $H \to Z\,Z$. This process has the following initial and final states,

$$|i\rangle = h^\dagger(\vec{k_1})\,|0\rangle, \quad |f\rangle = z^\dagger_{s_1}(\vec{k_2})\, z^\dagger_{s_2}(\vec{k_3})\,|0\rangle, \tag{15}$$

where we denote the Z operators with z, and the H operators with h. It is determined by this part of the interaction Lagrangian density

$$\mathscr{L}^{(1)} = \frac{vg^2}{4\cos(\theta_{\mathrm{W}})^2} Z_\alpha Z^\alpha \sigma \implies \mathscr{H}^{(1)} = -\frac{vg^2}{4\cos(\theta_{\mathrm{W}})^2} Z_\alpha Z^\alpha \sigma. \tag{16}$$

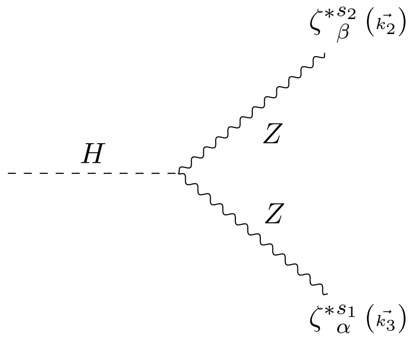

Figure 1: First-order electroweak process $H \to Z\,Z$.

It corresponds to the Feynman diagram in Figure 1. The S-matrix reads

$$\langle f|\, S^{(1)}\,|i\rangle = -\frac{(-\mathrm{i})vg^2}{4\cos\left(\theta_{\mathrm{W}}\right)^2}\,\langle f|\int \mathrm{d}^4x_1\,\mathrm{N}\left[Z_\alpha Z^\alpha \sigma\right]|i\rangle\,, \tag{17}$$

where $\mathrm{N}\left[\cdot\right]$ is the normal ordering operator. Now we expand it in terms of the fields

$$\begin{aligned}\langle f|\, S^{(1)}\,|i\rangle = {} & \frac{\mathrm{i}vg^2}{4\cos\left(\theta_{\mathrm{W}}\right)^2}\,\langle f|\int \mathrm{d}^4x_1\,\mathrm{N}\Bigg[\cdot \\ & \sum_{r=-1}^{1}\int\frac{\mathrm{d}^3p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}}\left(\zeta^r_\alpha\left(\vec{p_1}\right)z_r\left(\vec{p_1}\right)\mathrm{e}^{-\mathrm{i}p_1x_1}+\boxed{\zeta^{*r}_{\ \alpha}\left(\vec{p_1}\right)z^\dagger_r\left(\vec{p_1}\right)\mathrm{e}^{\mathrm{i}p_1x_1}}\right)\cdot \\ & \sum_{r'=-1}^{1}\int\frac{\mathrm{d}^3p_2}{(2\pi)^{3/2}\sqrt{E_{\vec{p_2}}}}\left(\zeta^\alpha_{r'}\left(\vec{p_2}\right)z_{r'}\left(\vec{p_2}\right)\mathrm{e}^{-\mathrm{i}p_2x_1}+\boxed{\zeta^{*\alpha}_{\ r'}\left(\vec{p_2}\right)z^\dagger_{r'}\left(\vec{p_2}\right)\mathrm{e}^{\mathrm{i}p_2x_1}}\right)\cdot \\ & \int\frac{\mathrm{d}^3p_3}{(2\pi)^{3/2}\sqrt{E_{\vec{p_3}}}}\left(\boxed{h\left(\vec{p_3}\right)\mathrm{e}^{-\mathrm{i}p_3x_1}}+h^\dagger\left(\vec{p_2}\right)\mathrm{e}^{\mathrm{i}p_3x_1}\right)\Bigg]|i\rangle\,,\end{aligned} \tag{18}$$

where we indicate the Z polarization vectors with ζ. We have eight possible terms. As we know from QED, only the terms annihilating the initial and final states matter, i.e., only the product of the boxed terms does not vanish. We are then left with

$$\begin{aligned}\langle f|\, S^{(1)}\,|i\rangle = {} & \frac{\mathrm{i}vg^2}{4\cos\left(\theta_{\mathrm{W}}\right)^2(2\pi)^{9/2}\sqrt{E_{\vec{p_1}}E_{\vec{p_2}}E_{\vec{p_3}}}}\sum_{r=-1}^{1}\sum_{r'=-1}^{1}\cdot \\ & \int \mathrm{d}^3p_1\mathrm{d}^3p_2\mathrm{d}^3p_3\,\zeta^{*r}_{\ \alpha}\left(\vec{p_1}\right)\zeta^{*\alpha}_{\ r'}\left(\vec{p_2}\right)\boxed{\langle f|\,z^\dagger_r\left(\vec{p_1}\right)z^\dagger_{r'}\left(\vec{p_2}\right)h\left(\vec{p_3}\right)|i\rangle}\cdot \\ & \int \mathrm{d}^4x_1\,\mathrm{e}^{\mathrm{i}x_1(p_1+p_2-p_3)}.\end{aligned} \tag{19}$$

We now work out the boxed term, containing the operators.

$$\langle f|\, z_r^\dagger(\vec{p_1})\, z_{r'}^\dagger(\vec{p_2})\, h(\vec{p_3})\, |i\rangle = \langle 0|\, z_{s_1}(\vec{k_2})\, z_{s_2}(\vec{k_3})\, z_r^\dagger(\vec{p_1})\, z_{r'}^\dagger(\vec{p_2})\, h(\vec{p_3})\, h^\dagger(\vec{k_1})\, |0\rangle\,. \tag{20}$$

We want to have all the annihilators to the right and all the creators on the left. We have to commute them, reminding the commutation relations,

$$\left[z_r(\vec{k})\,, z_s^\dagger(\vec{p})\right] = \delta_{rs}\delta(\vec{k}-\vec{p}). \tag{21}$$

We start,

$$\begin{aligned}&\langle 0|\, z_{s_1}(\vec{k_2}) \boxed{z_{s_2}(\vec{k_3})\, z_r^\dagger(\vec{p_1})}\, z_{r'}^\dagger(\vec{p_2}) \boxed{h(\vec{p_3})\, h_{r_1}^\dagger(\vec{k_1})}\, |0\rangle \\ =&\, \langle 0|\, z_{s_1}(\vec{k_2}) \boxed{z_r^\dagger(\vec{p_1})\, z_{s_2}(\vec{k_3}) + \delta_{s_2 r}\delta(\vec{k_3}-\vec{p_1})}\, z_{r'}^\dagger(\vec{p_2}) \boxed{h^\dagger(\vec{k_1})\, h(\vec{p_3}) + \delta(\vec{p_3}-\vec{k_1})}\, |0\rangle\,.\end{aligned} \tag{22}$$

It is convenient to consider operators on the right first, because if they annihilate the vacuum, the expression simplifies. In this case, the first term in the second box annihilate the vacuum, so it is zero. We then have two terms to consider, namely

$$\begin{aligned}&\langle 0| \boxed{z_{s_1}(\vec{k_2})\, z_r^\dagger(\vec{p_1})} \boxed{z_{s_2}(\vec{k_3})\, z_{r'}^\dagger(\vec{p_2})}\, |0\rangle\, \delta(\vec{p_3}-\vec{k_1}) \\ +&\, \langle 0| \boxed{z_{s_1}(\vec{k_2})\, z_{r'}^\dagger(\vec{p_2})}\, |0\rangle\, \delta(\vec{p_3}-\vec{k_1})\delta_{s_2 r}\delta(\vec{k_3}-\vec{p_1}).\end{aligned} \tag{23}$$

The commutation of the operators in the boxes gives,

$$\begin{aligned}&\langle 0| \boxed{z_r^\dagger(\vec{p_1})\, z_{s_1}(\vec{k_2}) + \delta_{s_1 r}\delta(\vec{k_2}-\vec{p_1})} \boxed{z_{r'}^\dagger(\vec{p_2})\, z_{s_2}(\vec{k_3}) + \delta_{r' s_2}\delta(\vec{k_3}-\vec{p_2})}\, |0\rangle\, \delta(\vec{p_3}-\vec{k_1}) \\ +&\, \langle 0| \boxed{z_{r'}^\dagger(\vec{p_2})\, z_{s_1}(\vec{k_2}) + \delta_{s_1 r'}\delta(\vec{k_2}-\vec{p_2})}\, |0\rangle\, \delta(\vec{p_3}-\vec{k_1})\delta_{s_2 r}\delta(\vec{k_3}-\vec{p_1}).\end{aligned} \tag{24}$$

In the first line, in the second box only the term with the deltas survive. Hence, of the two terms in the first box, again only the deltas survive. Same for the second line. We then have,

$$\delta_{s_1 r}\delta_{r' s_2}\delta(\vec{p_3}-\vec{k_1})\delta(\vec{k_2}-\vec{p_1})\delta(\vec{k_3}-\vec{p_2}) + \delta_{s_1 r'}\delta_{s_2 r}\delta(\vec{p_3}-\vec{k_1})\delta(\vec{k_2}-\vec{p_2})\delta(\vec{k_3}-\vec{p_1}). \tag{25}$$

Note that these deltas produce the topologically equivalent Feynman diagrams, as in QED. In this case, the polarization indices and the momenta $\vec{p_1}$ and $\vec{p_2}$ are switched in the two terms, meaning that we can exchange the two Z bosons in the Feynman diagram in Figure 1, since they are indistinguishable.

Now we can plug this expression in place of the box in (19). The sums over the polarizations go away due to the deltas, as well as the three integrals over the 3-momenta. Also, we integrate over x_1 taking $(2\pi)^{-4}$ from the prefactor, to get the Dirac delta that guarantees the conservation of 4-momentum.

$$\langle f|\, S^{(1)}\, |i\rangle = \frac{\mathrm{i}vg^2\delta\,(k_1 - k_2 - k_3)}{4\cos\,(\theta_{\mathrm{W}})^2(2\pi)^{1/2}\sqrt{E_{\vec{k_1}}E_{\vec{k_2}}E_{\vec{k_3}}}}\left[\zeta^{*s_1}_{\ \alpha}\,(\vec{k_3})\,\zeta^{*\alpha}_{\ s_2}\,(\vec{k_2}) + \zeta^{*s_2}_{\ \alpha}\,(\vec{k_2})\,\zeta^{*\alpha}_{\ s_1}\,(\vec{k_3})\right]. \tag{26}$$

As anticipated, the two terms in the square parentheses are equal, so we can sum them

$$\begin{aligned}\langle f|\, S^{(1)}\, |i\rangle &= \frac{\mathrm{i}vg^2\delta\,(k_1 - k_2 - k_3)}{2\cos\,(\theta_{\mathrm{W}})^2(2\pi)^{1/2}\sqrt{E_{\vec{k_1}}E_{\vec{k_2}}E_{\vec{k_3}}}}\zeta^{*s_1}_{\ \alpha}\,(\vec{k_3})\,\zeta^{*\alpha}_{\ s_2}\,(\vec{k_2}) \\ &= \frac{\delta\,(k_1 - k_2 - k_3)}{(2\pi)^{1/2}\sqrt{E_{\vec{k_1}}E_{\vec{k_2}}E_{\vec{k_3}}}}\zeta^{*s_1}_{\ \alpha}\,(\vec{k_3})\,\zeta^{*s_2}_{\ \beta}\,(\vec{k_2})\boxed{\frac{\mathrm{i}vg^2\,\eta^{\alpha\beta}}{4\cos\,(\theta_{\mathrm{W}})^2}}.\end{aligned} \tag{27}$$

The conjugate polarization vectors tell us that we have two vector bosons in the final state (same as a final photon in QED), whereas the box correspond to the Feynman rule for this kind of vertex [compare with eq. (B.11) in M&S].

(B.3) $W\,W^\dagger \to Z\,Z$**.** First, note that

$$W_\mu = W^1_\mu - \mathrm{i}W^2_\mu = W^+_\mu\,, \qquad W^\dagger_\mu = W^1_\mu + \mathrm{i}W^2_\mu = W^-_\mu\,, \tag{28}$$

i.e., W_μ has a positive electric charge and $W^\dagger_\mu$ has a negative one. This can be seen in the following way. Consider (17.27) in M&S

$$\frac{\mathcal{Q}}{e} = \mathcal{Y} + \mathcal{I}^W_3\,, \tag{29}$$

with $\mathcal{Y}$ hypercharge and $\mathcal{I}^W_3$ the isospin charge (the normal style for the operators, whereas the calligraphic style is used for the charges, i.e., the eigenvalues of the operators). Now, the W bosons have $\mathcal{Y} = 0$ by definition, hence their normalized electric charge is equal to their isospin charge. By definition, the isospin charge of the W-bosons is defined by the commutation relations of the associated charges with the operator I^W_3,

$$\left[I^W_3, I^W_i\right] = \mathcal{I}^W_i\, I^W_i\,. \tag{30}$$

From (28) we can define the currents $J^\pm_\mu$ and the associated charges $\mathcal{I}^W_\pm$, as it is described in Section 17.2 in M&S, and we discussed in the previous tutorials. Then, we know that

$$\left[I^W_3, I^W_\pm\right] = \pm I^W_\pm\,. \tag{31}$$

This implies that $W^+_\mu = W_\mu$ has $\mathcal{Q}/e = \mathcal{I}^W_3 = 1$, and $W^-_\mu = W^\dagger_\mu$ has $\mathcal{Q}/e = \mathcal{I}^W_3 = -1$. Therefore, these are the electric charge eigenstates.

We are going to use the electric charge eigenstates, rather than the mass eigenstates. Hence, we now define their annihilation and creation operators in terms of those of the mass eigenstates. This allows to write explicitly the field expressions for W_μ^+, W_μ^- as well. The mass eigenstates are

$$W_\alpha^1 = \sum_{r=-1}^{1} \int \frac{\mathrm{d}^3 p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}} \left(\omega_\alpha^r (\vec{p_1}) \overset{1}{w}_r (\vec{p_1}) \, \mathrm{e}^{-\mathrm{i} p_1 x_1} + \omega^{*r}_{\ \alpha} (\vec{p_1}) \overset{1}{w}{}_r^\dagger (\vec{p_1}) \, \mathrm{e}^{\mathrm{i} p_1 x_1} \right), \tag{32a}$$

$$W_\alpha^2 = \sum_{r=-1}^{1} \int \frac{\mathrm{d}^3 p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}} \left(\omega_\alpha^r (\vec{p_1}) \overset{2}{w}_r (\vec{p_1}) \, \mathrm{e}^{-\mathrm{i} p_1 x_1} + \omega^{*r}_{\ \alpha} (\vec{p_1}) \overset{2}{w}{}_r^\dagger (\vec{p_1}) \, \mathrm{e}^{\mathrm{i} p_1 x_1} \right), \tag{32b}$$

where the polarization vectors are the same, since both field are spin-1 massive fields. The charge eigenstates (28) are

$$\begin{aligned} W_\alpha^+ = W_\alpha = \sum_{r=-1}^{1} \int \frac{\mathrm{d}^3 p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}} & \Big[\omega_\alpha^r (\vec{p_1}) \left(\overset{1}{w}_r (\vec{p_1}) - \mathrm{i} \overset{2}{w}_r (\vec{p_1}) \right) \mathrm{e}^{-\mathrm{i} p_1 x_1} \\ & + \omega^{*r}_{\ \alpha} (\vec{p_1}) \left(\overset{1}{w}{}_r^\dagger (\vec{p_1}) - \mathrm{i} \overset{2}{w}{}_r^\dagger (\vec{p_1}) \right) \mathrm{e}^{\mathrm{i} p_1 x_1} \Big], \end{aligned} \tag{33a}$$

$$\begin{aligned} W_\alpha^- = W_\alpha^\dagger = \sum_{r=-1}^{1} \int \frac{\mathrm{d}^3 p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}} & \Big[\omega_\alpha^r (\vec{p_1}) \left(\overset{1}{w}_r (\vec{p_1}) + \mathrm{i} \overset{2}{w}_r (\vec{p_1}) \right) \mathrm{e}^{-\mathrm{i} p_1 x_1} \\ & + \omega^{*r}_{\ \alpha} (\vec{p_1}) \left(\overset{1}{w}{}_r^\dagger (\vec{p_1}) + \mathrm{i} \overset{2}{w}{}_r^\dagger (\vec{p_1}) \right) \mathrm{e}^{\mathrm{i} p_1 x_1} \Big]. \end{aligned} \tag{33b}$$

Now we can define the complex operators

$$\overset{-}{w}_r (\vec{p}) := \overset{1}{w}_r (\vec{p}) - \mathrm{i} \overset{2}{w}_r (\vec{p}), \qquad \overset{+}{w}_r (\vec{p}) := \overset{1}{w}_r (\vec{p}) + \mathrm{i} \overset{2}{w}_r (\vec{p}), \tag{34a}$$

and express a quantum state in their terms—we are going to do that with the initial state of our process.

This process has the following initial and final states,

$$|i\rangle = \overset{-}{w}{}_{r_1}^\dagger (\vec{k_1}) \overset{+}{w}{}_{r_2}^\dagger (\vec{k_2}) |0\rangle, \quad |f\rangle = z_{s_1}^\dagger (\vec{k_3}) z_{s_2}^\dagger (\vec{k_4}) |0\rangle. \tag{35}$$

It corresponds to the Feynman diagram in Figure 2, and it is determined by the following part of the interaction Lagrangian density,

$$\mathscr{L}^{(1)} = g^2 \cos(\theta_\mathrm{W})^2 \left(W_\alpha W_\beta^\dagger Z^\alpha Z^\beta - W_\beta W^{\dagger\beta} Z_\alpha Z^\alpha \right), \quad \mathscr{H}^{(1)} = -\mathscr{L}^{(1)}. \tag{36}$$

The procedure is the same as the previous case. The S-matrix reads

$$\langle f| S^{(1)} |i\rangle = -\mathrm{i} \left(-g^2 \cos(\theta_\mathrm{W})^2 \right) \langle f| \int \mathrm{d}^4 x_1 \, \mathrm{N} \left[W_\alpha W_\beta^\dagger Z^\alpha Z^\beta - W_\beta W^{\dagger\beta} Z_\alpha Z^\alpha \right] |i\rangle, \tag{37}$$

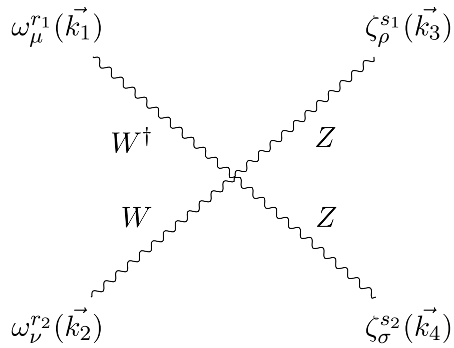

Figure 2: First-order electroweak process $W\,W^{\dagger} \to Z\,Z$.

which, in terms of the fields, becomes

$$
\begin{aligned}
\langle f|\, S^{(1)}\,|i\rangle = \mathrm{i}g^2\cos\left(\theta_{\mathrm{W}}\right)^2 \langle f| \int \mathrm{d}^4x_1 \cdot \mathrm{N}\Bigg[& \\
& \sum_{r=-1}^{1}\int \frac{\mathrm{d}^3p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}}\left(\boxed{\omega^{r}_{\alpha}\left(\vec{p_1}\right)\bar{w}_r\left(\vec{p_1}\right)\mathrm{e}^{-\mathrm{i}p_1x_1}} + \omega^{*r}_{\ \alpha}\left(\vec{p_1}\right)\overset{+}{w}^{\dagger}_r\left(\vec{p_1}\right)\mathrm{e}^{\mathrm{i}p_1x_1}\right)\cdot \\
& \sum_{r'=-1}^{1}\int \frac{\mathrm{d}^3p_2}{(2\pi)^{3/2}\sqrt{E_{\vec{p_2}}}}\left(\omega^{*r'}_{\beta}\left(\vec{p_2}\right)\overset{-}{w}^{\dagger}_{r'}\left(\vec{p_2}\right)\mathrm{e}^{\mathrm{i}p_2x_1} + \boxed{\omega^{r'}_{\beta}\left(\vec{p_2}\right)\overset{+}{w}_{r'}\left(\vec{p_2}\right)\mathrm{e}^{-\mathrm{i}p_2x_1}}\right)\cdot \\
& \sum_{s=-1}^{1}\int \frac{\mathrm{d}^3p_3}{(2\pi)^{3/2}\sqrt{E_{\vec{p_3}}}}\left(\zeta^{\alpha}_{s}\left(\vec{p_3}\right)z_s\left(\vec{p_3}\right)\mathrm{e}^{-\mathrm{i}p_3x_1} + \boxed{\zeta^{*\alpha}_{\ \ s}\left(\vec{p_3}\right)z^{\dagger}_s\left(\vec{p_3}\right)\mathrm{e}^{\mathrm{i}p_3x_1}}\right)\cdot \\
& \sum_{s'=-1}^{1}\int \frac{\mathrm{d}^3p_4}{(2\pi)^{3/2}\sqrt{E_{\vec{p_4}}}}\left(\zeta^{\beta}_{s'}\left(\vec{p_4}\right)z_{s'}\left(\vec{p_4}\right)\mathrm{e}^{-\mathrm{i}p_4x_1} + \boxed{\zeta^{*\beta}_{\ \ s'}\left(\vec{p_4}\right)z^{\dagger}_{s'}\left(\vec{p_4}\right)\mathrm{e}^{\mathrm{i}p_4x_1}}\right) \\
& -\sum_{r=-1}^{1}\int \frac{\mathrm{d}^3p_1}{(2\pi)^{3/2}\sqrt{E_{\vec{p_1}}}}\left(\boxed{\omega^{r}_{\beta}\left(\vec{p_1}\right)\bar{w}_r\left(\vec{p_1}\right)\mathrm{e}^{-\mathrm{i}p_1x_1}} + \omega^{*r}_{\ \beta}\left(\vec{p_1}\right)\overset{+}{w}^{\dagger}_r\left(\vec{p_1}\right)\mathrm{e}^{\mathrm{i}p_1x_1}\right)\cdot \\
& \sum_{r'=-1}^{1}\int \frac{\mathrm{d}^3p_2}{(2\pi)^{3/2}\sqrt{E_{\vec{p_2}}}}\left(\omega^{*\beta r'}\left(\vec{p_2}\right)\overset{-}{w}^{\dagger}_{r'}\left(\vec{p_2}\right)\mathrm{e}^{\mathrm{i}p_2x_1} + \boxed{\omega^{\beta r'}\left(\vec{p_2}\right)\overset{+}{w}_{r'}\left(\vec{p_2}\right)\mathrm{e}^{-\mathrm{i}p_2x_1}}\right)\cdot \\
& \sum_{s=-1}^{1}\int \frac{\mathrm{d}^3p_3}{(2\pi)^{3/2}\sqrt{E_{\vec{p_3}}}}\left(\zeta^{s}_{\beta}\left(\vec{p_3}\right)z_s\left(\vec{p_3}\right)\mathrm{e}^{-\mathrm{i}p_3x_1} + \boxed{\zeta^{*s}_{\ \ \beta}\left(\vec{p_3}\right)z^{\dagger}_s\left(\vec{p_3}\right)\mathrm{e}^{\mathrm{i}p_3x_1}}\right)\cdot \\
& \sum_{s'=-1}^{1}\int \frac{\mathrm{d}^3p_4}{(2\pi)^{3/2}\sqrt{E_{\vec{p_4}}}}\left(\zeta^{\beta}_{s'}\left(\vec{p_4}\right)z_{s'}\left(\vec{p_4}\right)\mathrm{e}^{-\mathrm{i}p_4x_1} + \boxed{\zeta^{*\beta}_{\ \ s'}\left(\vec{p_4}\right)z^{\dagger}_{s'}\left(\vec{p_4}\right)\mathrm{e}^{\mathrm{i}p_4x_1}}\right)\Bigg]\,|i\rangle\,.
\end{aligned}
\tag{38}
$$

We notice that the differences between the two terms are only the contractions between the polarization vectors, and not the operators. Therefore, the computation is simpler than it appears.

Only the term annihilating the initial and final state matter. It is the product of the boxed terms

$$
\begin{aligned}
\langle f|\, S^{(1)}\,|i\rangle = {} & \frac{\mathrm{i}g^2 \cos\left(\theta_{\mathrm{W}}\right)^2}{(2\pi)^6\sqrt{E_{\vec{p_1}}E_{\vec{p_2}}E_{\vec{p_3}}E_{\vec{p_4}}}} \sum_{r=-1}^{1}\sum_{r'=-1}^{1}\sum_{s=-1}^{1}\sum_{s'=-1}^{1} \cdot \\
& \int \mathrm{d}^3p_1\mathrm{d}^3p_2\mathrm{d}^3p_3\mathrm{d}^3p_4 \boxed{\langle f|\, z_s^\dagger\left(\vec{p_3}\right) z_{s'}^\dagger\left(\vec{p_4}\right) \bar{w}_r\left(\vec{p_1}\right) \overset{+}{w}_{r'}\left(\vec{p_2}\right)|i\rangle} \cdot \\
& \left[\omega_\alpha^r\left(\vec{p_1}\right)\omega_\beta^{r'}\left(\vec{p_2}\right)\zeta_s^{*\alpha}\left(\vec{p_3}\right)\zeta_{s'}^{*\beta}\left(\vec{p_4}\right) - \omega_\beta^r\left(\vec{p_1}\right)\omega^{r'\beta}\left(\vec{p_2}\right)\zeta_{s\alpha}^{*}\left(\vec{p_3}\right)\zeta_{s'}^{*\alpha}\left(\vec{p_4}\right)\right] \cdot \\
& \int \mathrm{d}^4x_1\, \mathrm{e}^{-\mathrm{i}x_1(p_1+p_2-p_3-p_4)}. \qquad (39)
\end{aligned}
$$

As usual, we play with the operators,

$$
\langle 0|\, z_{s_1}\left(\vec{k_3}\right) z_{s_2}\left(\vec{k_4}\right) z_s^\dagger\left(\vec{p_3}\right) z_{s'}^\dagger\left(\vec{p_4}\right) \bar{w}_r\left(\vec{p_1}\right) \boxed{\overset{+}{w}_{r'}\left(\vec{p_2}\right) \bar{w}_{r_1}^\dagger\left(\vec{k_1}\right)} \overset{+}{w}_{r_2}^\dagger\left(\vec{k_2}.\right)|0\rangle \qquad (40)
$$

The two operators in the box commute,

$$
\langle 0|\, z_{s_1}\left(\vec{k_3}\right) \boxed{z_{s_2}\left(\vec{k_4}\right) z_s^\dagger\left(\vec{p_3}\right)} z_{s'}^\dagger\left(\vec{p_4}\right) \boxed{\bar{w}_r\left(\vec{p_1}\right) \bar{w}_{r_1}^\dagger\left(\vec{k_1}\right)} \boxed{\overset{+}{w}_{r'}\left(\vec{p_2}\right) \overset{+}{w}_{r_2}^\dagger\left(\vec{k_2}\right)}|0\rangle\,. \qquad (41)
$$

Now we switch the operators in the three boxes, getting

$$
\langle 0|\, z_{s_1}\left(\vec{k_3}\right) \boxed{z_s^\dagger\left(\vec{p_3}\right) z_{s_2}\left(\vec{k_4}\right) + \delta_{s_2 s}\delta\left(\vec{k_4}-\vec{p_3}\right)} z_{s'}^\dagger\left(\vec{p_4}\right)|0\rangle\, \delta_{r_1 r}\delta\left(\vec{p_1}-\vec{k_1}\right)\delta_{r' r_2}\delta\left(\vec{p_2}-\vec{k_2}\right). \qquad (42)
$$

We have now two terms,

$$
\begin{aligned}
& \langle 0| \boxed{z_{s_1}\left(\vec{k_3}\right) z_s^\dagger\left(\vec{p_3}\right)} \boxed{z_{s_2}\left(\vec{k_4}\right) z_{s'}^\dagger\left(\vec{p_4}\right)} |0\rangle\, \delta_{r_1 r}\delta\left(\vec{p_1}-\vec{k_1}\right)\delta_{r' r_2}\delta\left(\vec{p_2}-\vec{k_2}\right) \\
& + \langle 0| \boxed{z_{s_1}\left(\vec{k_3}\right) z_{s'}^\dagger\left(\vec{p_4}\right)} |0\rangle\, \delta_{s_2 s}\delta\left(\vec{k_4}-\vec{p_3}\right)\delta_{r_1 r}\delta\left(\vec{p_1}-\vec{k_1}\right)\delta_{r' r_2}\delta\left(\vec{p_2}-\vec{k_2}\right). \qquad (43)
\end{aligned}
$$

When switching the operators in the box, only the terms with the deltas survive, because the others always have an annihilator acting on the vacuum state. Hence we finally get,

$$
\begin{aligned}
& \delta_{s_1 s}\delta\left(\vec{k_3}-\vec{p_3}\right)\delta_{s_2 s'}\delta\left(\vec{k_4}-\vec{p_4}\right)\delta_{r_1 r}\delta\left(\vec{p_1}-\vec{k_1}\right)\delta_{r' r_2}\delta\left(\vec{p_2}-\vec{k_2}\right) \\
& +\delta_{s_1 s'}\delta\left(\vec{p_4}-\vec{k_3}\right)\delta_{s_2 s}\delta\left(\vec{k_4}-\vec{p_3}\right)\delta_{r_1 r}\delta\left(\vec{p_1}-\vec{k_1}\right)\delta_{r' r_2}\delta\left(\vec{p_2}-\vec{k_2}\right). \qquad (44)
\end{aligned}
$$

We insert these delta inside the box in (39), and we get

$$
\langle f|\, S^{(1)}\,|i\rangle = \frac{\mathrm{i}g^2 \cos\left(\theta_{\mathrm{W}}\right)^2}{(2\pi)^2\sqrt{E_{\vec{k_1}}E_{\vec{k_2}}E_{\vec{k_3}}E_{\vec{k_4}}}}\delta\left[(k_1+k_2)-(k_3+k_4)\right] \cdot
$$

$$
\left[\omega_{\alpha}^{r_1}(\vec{k_1})\,\omega_{\beta}^{r_2}(\vec{k_2})\left(\zeta_{s_1}^{*\alpha}(\vec{k_3})\,\zeta_{s_2}^{*\beta}(\vec{k_4})+\zeta_{s_2}^{*\alpha}(\vec{k_4})\,\zeta_{s_1}^{*\beta}(\vec{k_3})\right)\right.
\left.-2\,\omega_{\beta}^{r_1}(\vec{k_1})\,\omega^{r_2\beta}(\vec{k_2})\,\zeta_{s_1\alpha}^{*}(\vec{k_3})\,\zeta_{s_2}^{*\alpha}(\vec{k_4})\right]. \tag{45}
$$

This expression can be rewritten as,

$$
\begin{aligned}
\langle f|\,S^{(1)}\,|i\rangle &= \frac{\mathrm{i}g^2\cos(\theta_{\mathrm{W}})^2}{(2\pi)^2\sqrt{E_{\vec{k_1}}E_{\vec{k_2}}E_{\vec{k_3}}E_{\vec{k_4}}}}\delta\left[(k_1+k_2)-(k_3+k_4)\right]\cdot \\
&\omega_{\mu}^{r_1}(\vec{k_1})\,\omega_{\nu}^{r_2}(\vec{k_2})\,\zeta_{s_1\rho}^{*}(\vec{k_3})\,\zeta_{s_2\sigma}^{*}(\vec{k_4})\left[\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}-2\,\eta^{\mu\nu}\eta^{\rho\sigma}\right] \\
&= \frac{1}{(2\pi)^2\sqrt{E_{\vec{k_1}}E_{\vec{k_2}}E_{\vec{k_3}}E_{\vec{k_4}}}}\delta\left[(k_1+k_2)-(k_3+k_4)\right]\cdot \\
&\omega_{\mu}^{r_1}(\vec{k_1})\,\omega_{\nu}^{r_2}(\vec{k_2})\,\zeta_{s_1\rho}^{*}(\vec{k_3})\,\zeta_{s_2\sigma}^{*}(\vec{k_4})\cdot \\
&\boxed{\mathrm{i}g^2\cos(\theta_{\mathrm{W}})^2\left[\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}-2\,\eta^{\mu\nu}\eta^{\rho\sigma}\right]}.
\end{aligned} \tag{46}
$$

Compare with eq. (B.3) in M&S.

3 The discovery of the Higgs boson

Quoting from the CERN website (click here):

> On 4 July 2012, the ATLAS and CMS experiments at CERN's Large Hadron Collider announced they had each observed a new particle in the mass region around 126 GeV. This particle is consistent with the Higgs boson predicted by the Standard Model. The Higgs boson, as proposed within the Standard Model, is the simplest manifestation of the Brout–Englert–Higgs mechanism. Other types of Higgs bosons are predicted by other theories that go beyond the Standard Model.
>
> On 8 October 2013 the Nobel prize in physics was awarded jointly to François Englert and Peter Higgs "for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider."

Unfortunately, Brout passed away before the award, and the Nobel prize cannot be awarded posthumously. That is why he is not mentioned.

After that, many more measurements have been done at the LHC, and all of them are consistent with the predictions of the standard electroweak theory (and with QCD).

4 Mention of the hierarchy problem

The mass of the Higgs boson gets large quantum corrections when considering high-order diagrams. These corrections are proportional to the squared masses of the particles involved in the diagrams, the heaviest (known) one being the top quark, with $m_t \sim 173.5\text{GeV}$. It is a common belief that the Standard Model is incomplete, and that there should be other particles with higher masses. Then, the Higgs' mass would get corrections from them too. Since gravity is not quantized yet, it is expected that new very massive particles should be produced at very high energies, close to the quantum gravity energy scale, the Planck mass $m_{\text{Pl}} \approx 1.22 \times 10^{19}\text{GeV}$. Then, they would contribute to the Higgs' mass through (enormous) quantum corrections.

Since the Higgs mass is $\sim 126\text{GeV}$, the quantum corrections should cancel between themselves with an incredibly high precision, which is not inconsistent theoretically, but it is believed to be *unnatural*. This fine-tuning problem is called the "hierarchy problem". It asks why the Higgs' mass is so lower than the quantum gravity energy scale, since, in the Standard Model, it gets corrections from *any*, known and unknown, particles in the model.

Note that, if the top quark was the highest mass particle in the theory—it is the highest mass *known* particle in the Standard Model, but it is believed there are heavier, still unknown, ones—then there would not be a hierarchy problem.

Quoting from Section 22.6.1, 22.6.2 in M.D. Schwartz.,*Quantum Field Theory and the Standard Model*, Cambridge University Press, 2014. ISBN: 9781107034730:

> [...] indirect evidence for the mass of the Higgs boson came from precision measurements of the W and Z masses and other electroweak parameters. As will be shown in Chapter 31, these get finite radiative corrections from loops involving quarks, most notably the top quark, and the Higgs. The on-shell pole mass for the top quark is $m_t \sim 173.5\text{GeV}$ while its $\overline{\text{MS}}$ mass is $m_t \sim 165.6\text{GeV}$ [Particle Data Group (Beringer et al.), 2012]. This 5% difference comes from loops involving gluons. For these calculations one should also use the $\overline{\text{MS}}$ mass for the Higgs bosons, which differs from the experimentally measured pole mass due primarily to the loop we just calculated involving the top quark.
>
> [...]
>
> Although the difference is finite, as $M \to \infty$ [the mass of the fermion in the diagrams, Ed.] the difference grows very large. Indeed, the difference is sensitive to particles much heavier than the mass of the scalar. Although the result is finite, heavy par-

ticles are not decoupling. In this way, the scalar mass is UV sensitive.

[...]

We saw that although the scalar mass gets quadratically divergent corrections, for example from a fermion loop, these divergences can be removed with counterterms. The resulting physical pole mass must be determined from experiment as a renormalization condition. It does not get corrections at any order in perturbation theory, since by definition it is the physical value of the mass. However, we saw that there can be a large difference between the pole mass and the $\overline{\text{MS}}$ mass for a scalar. In particular, the difference in the squares of these masses is proportional to the square of the mass of any fermion that couples to the scalar. Since heavy fermions do not decouple, the scalar mass is UV sensitive.

[...]

Fine-tuning is a sensitivity of physical observables (the pole mass) to variation of parameters in the theory. That the Higgs mass is so much smaller than the Planck scale (or some other scale where the UV completion for the Standard Model might live) is called the hierarchy problem. It is a problem with the theoretical concept of naturalness, which says that all parameters in a fundamental theory should be of order 1.
